Magnetothermoelectric effects in semiconductor systems

R. Fletcher

Physics Department, Queen's University, Kingston, Ontario, Canada, K7L 3N6.

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Abstract

This topical review examines work on the magnetothermoelectric properties of semiconductors that has appeared since the major review by Gallagher and Butcher in 1992. The focus is on the field dependence of the thermopower of degenerate 2D and 3D systems, both diffusion and phonon drag. The main conclusion is that the experimental observations on electrons, holes, two-carrier systems and composite fermions are all consistent with physical models and predictions based on the Boltzmann equation. A brief review of the possible effect of weak localization on phonon drag is also given.

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I. INTRODUCTION

A comprehensive review of work on low-dimensional systems up to 1992 was provided by Gallagher and Butcher[1], and regular reference will be made to this. The present article is more restricted in scope and focuses on recent developments in the magnetothermoelectric properties of semiconductors. It is mainly concerned with 2D degenerate electron or hole gases, though some recent results on a 3D system will also be discussed. Advances since the earlier review include new data on the low-field diffusion thermopower and phonon drag, measurements on composite fermions, and some very recent new work on weak localization. New theoretical results for phonon drag, which augment the standard theoretical treatment, have provided a very useful physical picture.

Some general considerations concerning transport will first be introduced. Following this there will be sections on experimental techniques and the theoretical framework. The main section will comprise a review of recent results.

II. GENERAL CONSIDERATIONS

It is convenient to first deal with the phenomenological equations which relate the electrical and thermal current densities $\mathbf{J}$ and $\mathbf{U}$ to the applied electric field $\mathbf{E}$ and temperature gradient ∇T, via four coefficients, i.e.,

$$\mathbf{J} = \overleftrightarrow{\sigma}\mathbf{E} - \overleftrightarrow{\epsilon}\nabla T \quad (1)$$

$$\mathbf{U} = \overleftrightarrow{\pi}\mathbf{E} - \overleftrightarrow{\lambda}\nabla T \quad (2)$$

The four coefficients are responsible for all the usual transport properties of materials. They are usually treated as scalars when a magnetic field is not present, but in a magnetic field $\mathbf{B}$ they all become tensors. The coefficients are the conductivity $\overleftrightarrow{\sigma}$, the thermoelectric tensor $\overleftrightarrow{\epsilon}$, the Peltier tensor $\overleftrightarrow{\pi}$, and the thermal conductivity $\overleftrightarrow{\lambda}$. Unless otherwise stated, the field will be assumed to be along z and the sample will be taken to be isotropic in the xy plane, with the current along x. The main case of interest will be that with $\mathbf{B}$ perpendicular to the sample, whether it be a 2D or 3D sample, so that, in general, there are two tensor components in each coefficient that are of interest. However, the situation is even simpler in practice because the thermal conductivity is dominated by phonon conduction in semiconductors so that only λ_{xx} is relevant, say λ, with $\lambda_{xy} \approx 0$. In addition, the Kelvin-Onsager relations[2] give $\overleftrightarrow{\pi}(\mathbf{B}) = T\overleftrightarrow{\epsilon}^{\dagger}(-\mathbf{B})$, where $\dagger$ represents the transpose. Thus there are only two relevant coefficients for each of $\overleftrightarrow{\sigma}$, $\sigma_{xx} = \sigma_{yy}$ and $\sigma_{xy} = -\sigma_{yx}$, and $\overleftrightarrow{\epsilon}$, $\epsilon_{xx} = \epsilon_{yy}$ and $\epsilon_{xy} = -\epsilon_{yx}$, and it is ϵ_{ij} that are of interest here.

In principle one could obtain the components of $\overleftrightarrow{\epsilon}$ by measuring the relevant Peltier coefficients, but in practice virtually all experimental results are obtained by measuring the components of the thermopower tensor, defined by $\mathbf{E} = \overleftrightarrow{S}\nabla T$ under the condition $\mathbf{J} = 0$. Clearly, $\overleftrightarrow{S} = \overleftrightarrow{\sigma}^{-1}\overleftrightarrow{\epsilon} = \overleftrightarrow{\rho}\overleftrightarrow{\epsilon}$, where $\overleftrightarrow{\rho}$ is the resistivity tensor. It is also necessary to specify a transverse condition for the measurement of $\overleftrightarrow{S}$. The only practical condition is probably $U_y = 0$. For semiconductors this is almost equivalent to $\nabla T_y = 0$ because of the dominance of the phonon thermal conductivity. (This is not so for bulk metals). With this one finds the two components of S to be given by

$$S_{xx} = E_x/\nabla T_x = \rho_{xx}\epsilon_{xx} + \rho_{xy}\epsilon_{yx} \tag{3}$$

$$S_{yx} = E_y/\nabla T_x = \rho_{yx}\epsilon_{xx} + \rho_{yy}\epsilon_{yx} \tag{4}$$

S_{xx} is the longitudinal thermopower, or just the thermopower, and S_{yx} is the Nernst-Ettingshausen (NE) coefficient, or the transverse thermopower. Strictly speaking, $\mathbf{E}$ should be replaced by the gradient of the electrochemical potential in Eq. (1), but the chemical potential part is not detected in experimental systems.[2–4]

The above considerations are rather general, but they are necessary because they indicate under what conditions the measurements should be made, and also what must be calculated for comparison with the experiments.

III. EXPERIMENTAL TECHNIQUES

Details of techniques to measure S_{ij} over a wide range of temperatures ($\leq 0.1\,\mathrm{K}$ to about $200\,\mathrm{K}$) and in magnetic fields have been given in various papers.[5–8] Apart from electrical contacts to the sample to measure potential difference, a means of measuring ∇T_x is required. Some early experiments used thermocouples, but these are a poor choice for use in magnetic fields. The temperature dependence of the resistivity of the sample itself has also been used with great success,[5] but again this is not suitable with a magnetic field. Commercially available surface-mount resistors from a variety of manufacturers, e.g. Dale, Philips and Siemens, attached to the sample with a small dab of epoxy, are well suited to this application and, as a bonus, their cost is negligible. They are very reproducible on successive cooldowns and are either insensitive to magnetic field or are only weakly sensitive. Those types that are field sensitive appear to be accurately matched in their field dependence. Any particular resistance value at room temperature is typically capable of covering a temperature range of more than a decade, the higher the nominal value, the higher the temperature at which it is useful. Bayot et al.[9] have used carbon paint, which is also an excellent choice though it is only practical when the sample is non-conducting, i.e., for insulating substrates with 2DEGs.

For an ideally matched pair of resistance thermometers, the difference in their resistances is approximately proportional to their temperature difference. Because of this it is more accurate and sensitive to measure the resistance of one of the thermometers, and the difference in resistance between the pair, rather than measuring the two resistances independently. This is particularly so when the temperature difference is small. Because the advantages of this technique do not seem to be widely known, a brief outline of the procedure will be given.

Fig. 1 shows the resistance versus temperature for a pair of resistors R_a and R_b that are not perfectly matched. The calibration measures R_a and $\Delta R = R_a - R_b$ as a function of T (and B if necessary). Smooth fits are made to ΔR versus R_a, and to R_b versus T. The accuracy of ΔT is essentially determined by the accuracy with which the gradient of R_b versus T can be obtained. This means that high power polynomials must be avoided in the fit.

With a temperature gradient present so that R_a is at T_2 and R_b is at T_1, $R_a(T_2)$ and $\Delta R = R_a(T_2) - R_b(T_1)$ are measured. From these $R_b(T_1)$ is immediately obtained. Using $R_a(T_2)$ and the calibration of ΔR versus R_a, one calculates $R_a(T_2) - R_b(T_2)$ and hence $R_b(T_2)$. Finally $R_b(T_2)$ and $R_b(T_1)$ and the calibration of $R_b(T)$ are used to find the temperature difference. Notice that the method does not use calibrations of the two resistors versus temperature, which is more susceptible to slight inaccuracies, particularly when the temperature gradient is small. As a simple test one can determine ΔT in the absence of a temperature gradient; the errors and noise usually result in $\Delta T \sim 0.2\,\mathrm{mK}$ in the ^{3}He and ^{4}He range, though there is some degradation at high magnetic fields. The technique is relatively immune to temperature drift.

It is important to emphasize that the conditions on the measurement of $\overleftrightarrow{S}$ discussed in Sec. II imply that the sample must be thermally isolated from its surroundings, except for contact at one end to the cold sink. This can only be ensured by making the measurements in high vacuum and using leads to the sample and thermometers made of a material with a high thermal resistance compared to both the sample and any thermal joints. Manganin is a good choice[10] because of its high thermal resistivity combined with very low thermopower.

Because of the problems that can arise with the thermometry, it has become standard practice to measure the sample thermal conductivity, λ, in Eq. (1) as a test of the techniques. At low temperatures, typically $T < 4\,\mathrm{K}$, the phonon conductivity of most semiconductor samples is dominated by boundary scattering for which $\lambda \propto T^3$. Under these conditions one can make a reasonable estimate of the expected value of the coefficient of T^3 using the calculations of Wybourne et al.[11] and McCurdy[12] for comparison with the experimental results. For checks over a wider temperature range there are often relevant data on thermal conductivity available in the literature. The results also enable the mean free path of the phonons, Λ, to be found; this is required in analysing phonon drag, as will be seen in the next section.

IV. THEORETICAL RESULTS

The thermopower tensor $\overleftrightarrow{\epsilon}$ has two additive contributions, diffusion $\overleftrightarrow{\epsilon^d}$ and phonon drag $\overleftrightarrow{\epsilon^g}$, which give rise to corresponding contributions $\overleftrightarrow{S^d}$ and $\overleftrightarrow{S^g}$ in $\overleftrightarrow{S}$. With a magnetic field, but ignoring weak localization, there are three regions of interest. At low fields one first sees effects due to the classical cyclotron motion of the carriers. In the second region, provided the temperature is low enough, quantum oscillations in the density of states cause the appearance of oscillations in all the transport coefficients. Finally, at high enough fields and low enough temperatures, 2D carriers exhibit the quantum Hall or fractional quantum Hall effect. Of course, these regions overlap to varying extents depending on the system. The last stage does not occur in 3D systems. The expected behaviour of both diffusion and phonon drag in each of these regions will first be examined. This will provide a basis for discussion of the experimental data in the next section, where other theoretical results will also be mentioned.

A. Diffusion Effects

Although the first theoretical papers on the magnetothermoelectric properties of 2D systems dealt only with the diffusion component, and this mainly in the quantum Hall region, it proved to be difficult to obtain relevant experimental data for comparison. In fact, even now an unambiguous experimental demonstration of diffusion thermopower in the quantum Hall region is not available. However, results at low fields in some particular systems have been obtained, as will be discussed in Section V A. The theoretical models that have proved useful in understanding this work will now be introduced.

Almost all the experimental work has been on samples with degenerate electron or hole gases in a temperature range where carrier scattering by the impurities is dominant. Ignoring quantum oscillations for the moment, when the scattering is elastic the diffusion thermopower is given by the well known Mott result which is arrived at as follows. In the presence of electric and magnetic fields and a temperature gradient, the Boltzmann equation gives a solution for the electron distribution from which one finds that the conductivity $\overleftrightarrow{\sigma}$ can be expressed in the form[1,2]

$$\overleftrightarrow{\sigma} = \int_0^\infty d\varepsilon \left(-\frac{\partial f_o}{\partial \varepsilon} \right) \overleftrightarrow{\sigma}(\varepsilon) \tag{5}$$

where f_o is the Fermi distribution function. $\overleftrightarrow{\sigma}(\varepsilon)$ can be considered to be the tensor conductivity that would be measured at $T = 0$ as a function of electron energy ε and contains all the dynamical and scattering information for the electrons. The diffusion thermoelectric tensor is given by

$$\overleftrightarrow{\epsilon}^d = -\frac{1}{eT} \int_0^\infty d\varepsilon \left(-\frac{\partial f_o}{\partial \varepsilon} \right) (\varepsilon - \varepsilon_F) \overleftrightarrow{\sigma}(\varepsilon) \tag{6}$$

where e is the magnitude of the electronic charge. These results seem to be more general than the Boltzmann equation.[13] Assuming $\overleftrightarrow{\sigma}$ is a slowly varying function of ε and expanding $\partial f_o / \partial \varepsilon$ about ε_F, one finds that

$$\overleftrightarrow{\epsilon}^d = -L_o e T \left(\frac{\partial \overleftrightarrow{\sigma}}{\partial \varepsilon} \right)_{\varepsilon_F} \tag{7}$$

where L_o is the Sommerfeld value of the Lorenz number $\pi^2 k_B^2 / 3e^2$. Using the free electron results for the components of $\overleftrightarrow{\sigma}$ and substituting in Eqs. (4) and (3) gives

$$S_{xx}^d = -\frac{L_o e T}{\varepsilon_F} \left(1 + \frac{p}{1 + \beta^2} \right) \tag{8}$$

$$S_{yx}^d = -\frac{L_o e T}{\varepsilon_F} \left(\frac{p\beta}{1 + \beta^2} \right) \tag{9}$$

where $\beta = \omega_c \tau_t$, τ_t being the transport (momentum) lifetime that one obtains from the mobility. It is assumed that $(\partial \ln \tau / \partial \ln \varepsilon)_{\varepsilon_F} = p$ and $(\partial \ln n / \partial \ln \varepsilon)_{\varepsilon_F} = 1$, the latter being appropriate to 2D. In 3D $(\partial \ln n / \partial \ln \varepsilon)_{\varepsilon_F} = 3/2$ and this would replace the unity in Eq. (8). Kuleev et al.[14] and Zianni et al.[15] have given equivalent equations. The results reduce to the usual zero-field Mott relation $S = -L_o e T (\partial \ln \sigma / \partial \varepsilon)_{\varepsilon_F}$ when $\beta = 0$.

An interesting feature is implicit in these equations. It has long been known (see e.g. Ref 16) that, when $p = 0$, the zero-field thermopower represents the entropy per unit charge, i.e., $S^d = -\mathcal{S}/ne$ where $\mathcal{S}$ is the entropy of the n carriers per unit area or volume. This is valid for both degenerate and non-degenerate systems, and Eqn. (8) is clearly consistent with this (both in 2D and 3D). It is also seen that, in the high field limit as $\beta \to \infty$, $S^d_{xx} \to -\mathcal{S}/ne$ precisely, a result emphasized by Oberaztsov.[17] Interestingly, if S^d_{xx} for non-degenerate semiconductors with elastic scattering is evaluated, the same result is again found to hold. Thus at zero field for electrons in 3D one finds[2]

$$S^d = -\left(\frac{k_B}{e}\right)\left(\frac{5}{2} + p - \frac{\varepsilon_F}{k_B T}\right) \tag{10}$$

where ε_F is measured from the conduction band edge, i.e. it is a negative quantity. A similar relation holds for hole gases. (In 2D the factor 5/2 is replaced by 2). Further, one finds that the equation remains the same as $\beta \to \infty$ except that the factor p disappears and the result again becomes equivalent to -$\mathcal{S}/ne$. The fact that scattering effects in S^d_{xx} disappear as $\beta \to \infty$ suggests that the entropy result might be rather general in this limit, and perhaps valid for arbitrary degeneracy and inelastic scattering, but this problem does not seem to have been addressed in the literature. The connection with entropy re-emerges later in Section V A.

As the magnetic field increases, the Landau levels begin to be resolved. At low fields the broadening of the levels Γ is determined by the quantum lifetime τ_q and for quantum oscillations to become visible requires $\omega_c\tau_q \gtrsim 1$. A simultaneous requirement is that $\hbar\omega_c \gtrsim kT$. The basic physics is that the elastic electron-impurity (e-i) scattering probability $(1/\tau_{ei})$ of the carriers is modulated by the oscillations in the density of states. In 2D the effective number of electrons is also modulated,[18] though in 3D this seems to be a small effect, at least at not too high magnetic field. Havlová and Smrčka[19] have given results for the oscillatory part of S^d for 2D systems at low magnetic fields, but a straightforward method of handling this was first used by Young[20] for the case of bulk metals. Young's approach will be outlined because it is physically transparent and has been used to analyse recent experiments in both

2D and 3D systems.

At low fields it is assumed that the components of $\overleftrightarrow{\sigma}$ have oscillating parts superposed on the smooth semi-classical components given in Eq. (5), and write these as $\tilde{\sigma}_{ij}(\varepsilon) = A_{ij}\cos\phi(\varepsilon)$ as $T \to 0$, where $\phi(\varepsilon) = \phi(\varepsilon_F) + 2\pi(\varepsilon - \varepsilon_F)/\hbar\omega_c$ and the prefactors A_{ij} are functions that vary only slowly with ε. The integral can be evaluated[20] and yields the well known result that at finite temperatures $\tilde{\sigma}_{ij} = D(X)A_{ij}\cos\phi(\varepsilon_F)$ where $X = X/\sinh X$, and $X = 2\pi k^2T/\hbar\omega_c$. The broadening of the Fermi function strongly damps the oscillations when $kT \sim \hbar\omega_c$.

Proceeding in the same way with Eq. (6) one finds[20]

$$\tilde{\epsilon}^d_{ij} = -(\pi k/e)D'(X)A_{ij}\sin\phi(\varepsilon_F) \tag{11}$$

where $D'(X) = dD(X)/dX$. The phase shift between $\tilde{\sigma}_{ij}$ and $\tilde{\epsilon}_{ij}$ should be a clear indicator of these low field diffusion oscillations. A convenient way of writing the above result is

$$\tilde{\epsilon}^d_{ij} = -i\frac{\pi k}{e}\frac{D'(X)}{D(X)}\tilde{\sigma}_{ij} = \alpha\tilde{\sigma}_{ij} \tag{12}$$

where $i = \sqrt{-1}$ indicates the $\pi/2$ phase shift between $\tilde{\epsilon}_{ij}$ and $\tilde{\sigma}_{ij}$. Using this and Eqns. (4) and (3), $\tilde{S}_{ij}$ can be evaluated. For the present purposes the form of the resulting equations given in Ref. 7 is the most useful and is

$$\begin{aligned}\tilde{S}^d_{xx} &= -\alpha(\tilde{\rho}_{xx}\bar{\sigma}_{xx} + \tilde{\rho}_{xy}\bar{\sigma}_{yx})\\ &= \frac{\alpha}{1+\beta^2}\left(\frac{\tilde{\rho}_{xx}}{\bar{\rho}_{xx}} + \beta^2\frac{\tilde{\rho}_{yx}}{\bar{\rho}_{yx}}\right)\end{aligned} \tag{13}$$

$$\begin{aligned}\tilde{S}^d_{yx} &= -\alpha(\tilde{\rho}_{xx}\bar{\sigma}_{yx} + \tilde{\rho}_{yx}\bar{\sigma}_{yy})\\ &= \frac{\alpha\beta}{1+\beta^2}\left(\frac{\tilde{\rho}_{xx}}{\bar{\rho}_{xx}} + \frac{\tilde{\rho}_{yx}}{\bar{\rho}_{yx}}\right)\end{aligned} \tag{14}$$

where the smooth parts are indicated by a bar in these equations. The results assume free electron relations for the smooth parts of $\overleftrightarrow{\sigma}$, but are otherwise reasonably general and are valid for both 2D and 3D. Note that diffusion thermopower oscillations are damped by $D'(X)$ and Young[20] gives figures showing $D(X)$ and $D'(X)$ for comparison. It is clear from the derivation that the results can be considered as just another extension of the Mott relation quoted above. Indeed the phase difference between $\tilde{\sigma}_{ij}$ and $\tilde{\epsilon}^d_{ij}$ arises because of this connection.

The important point is that $\tilde{S}^d_{ij}$ and $\tilde{\rho}_{ij}$ are intimately related in this approach and the identification of the precise mechanism leading to the oscillations in ρ is not required. These results are basically equivalent to those given by Havlová and Smrčka[19] for the 2D case, though these authors used a particular model for $\tilde{\sigma}$ in their evaluation. However, the earlier theoretical results of Obraztsov[17] for $\tilde{S}^d_{xx}$ at high B, which were used to analyse early experimental data on 3D degenerate semiconductors, are inconsistent with them. This point will be mentioned again in Section V A.

This section concludes by mentioning the result for S^d_{xx} expected in the quantum Hall region for 2D systems. In this region the separation into quasi-classical and quantum oscillatory parts is no longer appropriate. A number of authors[1] considered this problem and found that, when $kT \ll \hbar\omega_c$, then S_{xx} oscillates between zero (when ε_F lies between the levels) to maxima given by

$$S^d_{xx} = -\frac{k_B \ln 2}{e(m+\frac{1}{2})} \tag{15}$$

when a Landau level is half full and m is the number of completely full Landau levels. This result is expected to be valid only for a sample with low disorder, and assumes $kT \gg \Gamma$ and that there is no spin splitting. Interestingly, this is just $-\mathcal{S}/ne$ again. It has also been shown to be consistent with the Mott relation,[21] but notice that these oscillations are in phase with those for ρ_{xx}, unlike the situation at low fields where a phase difference of $\pi/2$ occurs. For low disorder samples S^d_{yx} is expected to be small at high fields[1] but the details will not be reproduced here.

B. Phonon Drag

The framework for understanding phonon drag $\overleftrightarrow{S}^g$ in 2D systems was established in a series of papers by Butcher's group at the University of Warwick.[1] The essence of the problem is that the phonons are not in equilibrium in the substrate, but preferentially flow down the temperature gradient ∇T_x. Because of the e-p interaction, carriers are dragged towards the colder end of the sample giving an extra contribution to the current and hence to ϵ.

Most of the theoretical work has involved the behaviour of phonon drag at zero field, though there have been extensions to high fields including the quantum Hall regions. These topics are well covered in Ref. 1 and it is not our intention to reexamine this material here. As far as we are aware, the only new theoretical paper dealing with S^g in the region where Landau levels must be taken into account is that of Fromhold et al.[22] This predicted $\epsilon^g_{xx} = 0$ which led led to difficulties in understanding the accompanying experimental data. The result may be due to the assumption of no disorder and it is probably not coincidental that one obtains the same result classically in the same limit (see Eq.16 below).

One might have hoped for progress in the low-field region where the basic physics is clearly similar to that for $\tilde{S}^d_{ij}$ in that the e-p momentum relaxation time τ_{ep} is modulated by oscillations in the density of states. (As shown below, S^g does not reflect the impurity momentum relaxation time τ_{ei}, at least not in the low field region). However, there is a great simplification in the low-field results for $\tilde{S}^d_{ij}$ due to the fact that impurity scattering is elastic. No similar simplification appears to be possible for $\tilde{S}^g_{ij}$.

Much of the new work has been concerned with the semi-classical region before quantum oscillations become important. The first major step in this direction was taken by Zianni et al.[15] who solved the Boltzmann equation and found that S^g_{xx} is independent of magnetic field, and $S^g_{yx} = 0$. The paper actually gives small correction terms for both S^g_{ij}, but subsequent work[23] has shown these to be negligible. Recently Miele et al[24] has confirmed this result in a way that provides a simple, but powerful, physical picture of phonon drag. Using a similar semi-classical formulation and Debye approximations they found that $\overleftrightarrow{\epsilon^g}$ can be written in the form

$$\overleftrightarrow{\epsilon^g}(B) = -\sum_s \frac{m^* v_s \Lambda_s}{eT\tau^s_{ep}} \overleftrightarrow{\sigma}(B) \tag{16}$$

where Λ_s is the phonon mean free path and v_s the velocity of the acoustic mode s. The result assumes impurity scattering is dominant, the gas is degenerate, and e-p scattering has no significant effect on the phonons. In the above expression, τ_{ep}^s represents the e-p momentum relaxation time for scattering by the mode s. (The e-p relaxation time as appears in the above expression for phonon drag is not precisely the same as that appropriate to the phonon limited mobility, except in the limit of a degenerate gas at low temperatures. The original paper should be consulted for details). The factor $1/\tau_{ep}$ reflects the rate at which momentum is pumped into the electron system by e-p scattering. This connection with τ_{ep} was not explicit in earlier theoretical results, though the physical content was the same.

A key point is that it does not matter whether the electrons gain momentum by e-p scattering or by an electric field, the contributions to the electric current are physically equivalent. Indeed, Miele et al showed that it is possible to define an effective electric field $\mathbf{E}_{ph}$ which would have exactly the same effect on the electrons as the applied temperature gradient acting through the e-p interaction. This is given by

$$\mathbf{E}_{ph} = \sum_s \frac{m^* v_s \Lambda_s}{eT\tau_{ep}^s} \nabla T \tag{17}$$

In other words, the current due to drag is $\mathbf{J}^g = \overleftrightarrow{\sigma}(B)\mathbf{E}_{ph}$. The electrons retain their memory of the momentum impulse for a time determined by e-i scattering, i.e., τ_{ei}, in exactly the same way as is appropriate to the conductivity. If there is a magnetic field, the electrons are subject to the Lorentz force and, in all respects, behave dynamically as if there was a real electric field present.

With $\overleftrightarrow{S}^g(B) = \overleftrightarrow{\sigma}^{-1}(B)\overleftrightarrow{\epsilon}^g(B)$, then

$$\overleftrightarrow{S}^g(B) = -\left(\sum_s \frac{v_s \Lambda_s}{\mu_{ep}^s T}\right)\overleftrightarrow{1} \tag{18}$$

where $\overleftrightarrow{1}$ is a unit matrix. Here the convenient notation $\mu_{ep}^s = e\tau_{ep}^s/m^*$ has been used; this would be the electron mobility if only e-p scattering by the mode s were present (but see the note above concerning possible differences). This equation is valid for both 2D and 3D degenerate semiconductors. It shows that S_{xx}^g is independent of magnetic field and that

$S^g_{yx} = 0$, both consistent with the results of Zianni et al.[15] It is an interesting, almost paradoxical, result that, even though e-i scattering must be dominant for Eq. (18) to be valid, τ_{ei} does not appear in the final expression and so S^g_{ij} should be independent of carrier mobility. The experimental data are in accord with this.

An equation similar to Eq. (18) was first derived by Herring[25] for non-degenerate semiconductors at zero field. It has been applied to degenerate 2D systems on a number of occasions and is discussed in Ref. 1. It was thought to be an approximation until it was empirically discovered to be exact for degenerate systems at low temperatures in an experimental paper by Tieke et al.[8]

There are other types of experiments which also probe phonon drag. Pulsed non-equilibrium phonons have been produced by heating thin films deposited on the substrate. The interaction of these phonons with the 2DEG is basically the same as with the thermopower, and has been detected in various ways.[26,27] The coupling between separated 2D electron and hole layers[28] can also be of the phonon drag type under certain conditions.[29]

V. RECENT RESULTS

A. Diffusion results

The progress made with the measurement of the diffusion component in a magnetic field is first examined. The obstacle to earlier progress in 2D systems is the fact that phonon drag is dominant over a wide temperature range. As discussed in Ref. 1, the early work regarding diffusion oscillations is unreliable.

The first systematic survey of the behaviour of the smooth parts of S_{ij} as a function of B were made by Fletcher et al.[30] using a low-mobility 2DEG in a δ-doped quantum well. At high temperatures ($T \gtrsim 100\,\mathrm{K}$) diffusion became visible and was analysed in terms of Eqs. (8) and (9). Reasonable, but not perfect, agreement was obtained.

The most striking experimental results are probably those of Tieke et al.[7] who examined the thermoelectric properties of 3D HgSe doped with Fe. The reason for the choice of this material was that relatively large quantities of Fe can be incorporated into the lattice, thereby producing a high density degenerate gas, while still maintaining a high carrier mobility. The zero field thermopower clearly shows the dominance of phonon drag over diffusion in this sample at low temperatures. Nevertheless, in the case of S_{yx} it was found that Eqs. (9) and (14) completely accounted for the observed behaviour of both the quantum oscillations and the smoothly varying background. (It was found that ρ_{yx} makes a negligible contribution in these calculations). This shows that $S^g_{yx} = 0$ to high precision, in agreement with Eq. (18), and also that the basic models for the field dependence of both the smooth and oscillatory parts of S^d_{yx} are correct. The expected phase shift of $\pi/2$ between $\tilde{S}^d_{yx}$ and $\tilde{\rho}_{xx}$ was accurately reproduced in the experiments. Some typical data are shown in Fig. 2. The oscillations in ρ_{xx} have a rather complex behaviour in this sample, so it was a significant advantage to be able to predict $\tilde{S}^d_{yx}$ in terms of $\tilde{\rho}_{xx}$ without requiring a detailed model calculation for the behaviour of the oscillations.

At low temperatures, harmonic structure becomes very pronounced in both $\tilde{\rho}_{xx}$ and $\tilde{S}_{yx}$, but the calculations still reproduce the latter remarkably accurately.[7] To obtain such detailed agreement, it was necessary to Fourier analyse $\tilde{\rho}_{xx}$ into the fundamental and the first two harmonics, calculate the corresponding harmonic contributions to $\tilde{S}^d_{yx}$ using the appropriate phase shifts and damping, and sum them to obtain the resultant curves. It should be noted that there are no free parameters in the calculations for the oscillations.

The smooth background in S_{xx} was also fully accounted for by Eq. (8), implying that S^g_{xx} is independent of B as predicted by Eq. (18). Unfortunately, it was not possible to test Eq. (13) with regard to $\tilde{S}^d_{xx}$ because of the dominance of phonon drag which also shows oscillations due to the modulation of τ_{ep} by the oscillatory density of states. Earlier experiments on $\tilde{S}_{xx}$ in degenerate semiconductors, e.g., Ref. 31, ignored drag and analysed the oscillatory data in terms of the theoretical results for diffusion of Oberaztsov[17] who predicted that, at high fields, both the smooth and the oscillatory parts of S^d_{xx} are given by the entropy per unit charge, $-\mathcal{S}/ne$. In view of our comments in Section IV concerning the applicability of this result in many situations, this is a very appealing result. However, although the smooth part agrees with Eq. (8), the oscillatory part is inconsistent with Eq. (13). Further discussion concerning this discrepancy is available in Tieke et al.[7] It has not yet proved possible to distinguish which, if any, of the two predictions for $\tilde{S}^d_{xx}$ is correct for 3D degenerate semiconductors. In principle the two predictions are readily distinguished experimentally because entropy oscillations are in phase with density of states oscillations, and hence $\tilde{\rho}_{xx}$, but Eq. (13) predicts a $\pi/2$ phase difference.

The early experiments on bulk metals, initiated by Young[20] and extended by Fletcher,[32] were also completely consistent with the theory based on the same assumptions as outlined in Section IV, though it should be pointed out that the situation is rather different in metals. There the thermal conductivity is dominated by electrons and for most of the systems investigated the boundary condition $U_y = 0$ typically results in $\nabla T_y \gg \nabla T_x$. Experimentally, the situation is also simplified in metals because $\tilde{S}^g_{xx}$ is negligible. This is because oscillations in the density of states, and thus τ_{ep}, rarely exceed 1% in bulk metals. Incidentally, Eq. (16) is not valid in metals because it assumes that only N-processes (and not U-processes) are allowed in the e-p interaction and also that the phonons are not significantly affected by e-p scattering.

The first experiments aimed specifically at separating diffusion and drag oscillations in 2D were carried out by Fletcher et al.[33] on a GaAs/$Ga_{1-x}Al_xAs$ heterojunction, but were only partially successful. A sample was grown on a strongly p-doped substrate in order to reduce the phonon mean free path Λ and thereby phonon drag (cf. Eq. (18)). Again, S^g_{yx} provided the most convincing evidence that $\tilde{S}^d_{ij}$ had been observed but there were several problems. In particular, it is now known that S^g_{yx} is not precisely zero in 2D systems. This will be discussed in more detail below. Oscillations were seen in S_{yx} that had the correct amplitude dependence as a function of B, but their absolute amplitude was about a factor of two larger than predicted by theory and their phase was not clear. At that time Eq. (14) was not available and the predictions were made in terms of model results for the components of $\tilde{\rho}_{ij}$. There was no correspondence between theory and experiment for $\tilde{S}^d_{xx}$ and it seems likely that only $\tilde{S}^g_{xx}$ was actually being observed.

After these experiments had been completed, it became clear that GaAs/$Ga_{1-x}Al_xAs$ based samples were a poor choice for two reasons. First, in such samples it is known that the momentum lifetime, τ_t which equals τ_{ei} here, is typically an order of magnitude greater than the quantum lifetime τ_q as determined from the broadening of the Landau levels (at low fields $\Gamma\tau_q = \hbar/2$). Quantum oscillations become visible when $\omega_c\tau_q \approx 1$, but by this point $\beta = \omega_c\tau_t \gg 1$ so that $\tilde{S}^d_{yx}$, as predicted by Eq. (14), is severely reduced in magnitude. The situation is even worse for $\tilde{S}^d_{xx}$ according to Eq. (13). Second, samples based on piezoelectric materials such as GaAs have an extra contribution to e-p coupling due to the piezoelectric effect, over and above the deformation potential coupling exhibited by all materials. Piezoelectric coupling is particularly important at the lowest temperatures and dominates e-p scattering in GaAs heterojunctions below about 1 K. This has the effect of keeping phonon drag large to very low temperatures.

Both of these problems were overcome by the choice of a Si inversion layer for a later set of experiments. The measurements were made on a high mobility MOSFET for which piezoelectric coupling is absent so that drag is very small[34] by 1 K, and also $\tau_t \approx \tau_q$. As shown in Fig. 3, at low temperatures the experimental results[35] on $\tilde{S}_{yx}$ are essentially perfectly reproduced in both form and magnitude by Eq. (14). The same technique of dealing with the harmonic content, as was mentioned above with reference to the HgSe:Fe data, was also used here. The oscillatory part of S_{xx} was found to be about a factor of two larger than predicted by Eq. (13), but otherwise it has all the same features including phase and field dependence. Examples are shown in Fig. 4. Again, note that the calculations have no free parameters.

The smooth parts of S_{ij} are also shown in the same figures and the experimental data on S_{xx} is seen to agree well with S^d_{xx} as calculated by Eq. (8) at all temperatures. This is so even when S^g_{xx} is dominant, thus implying that S^g_{xx} is independent of B as predicted by Eq. (18). Finally, the smooth part of S^d_{yx} as predicted by Eq. (9) appears to be present, but is augmented by an extra component which is assumed to be phonon drag. In other words, the experiments suggested $S^g_{yx} \neq 0$ in contradiction with Eq. (18). A more detailed discussion of this point will be postponed until later. In practice there are again no free parameters in the calculated curves because p is obtained from the zero field diffusion thermopower.

Since the earlier review[1] was published, a major development in the understanding of the behaviour of 2D systems at very high magnetic fields has taken place. This involves the interpretation of the properties of 2DEGs in the fractional quantum Hall regime in terms of a new kind of quasiparticle, the composite fermion (CF). Many reviews are available on this topic, including a recent topical review in this journal.[36] Our interest is in the thermoelectric properties of CFs, and the key points one needs in order to follow the discussion given below are: (i) At even denominator filling factors ν of the Landau levels, e.g., $\frac{1}{2}$, $\frac{3}{2}$, $\frac{3}{4}$ etc., the system can be considered to be a collection of quasiparticles obeying Fermi statistics, each particle being an electron bound to an even number of flux quanta. (ii) At exactly these values of ν, say ν_0, the particles act as if the external magnetic field, say $B(\nu_0)$, is zero. They have a Fermi surface and, in general, behave like electrons at zero field though their effective mass is about an order of magnitude higher. (iii) As the field is increased or decreased away from $B(\nu_0)$ to some value B, the particles respond as if the applied field is $B - B(\nu_0)$, which can be either sign. (iv) The oscillations in the fractional regime are due to the Landau levels of the CFs, and thus correspond to the integer quantum Hall effect for these particles.

The general behaviour of the thermopower at even denominator filling factors is found to be consistent with the CF model, in that the observed features are very similar to those seen for electrons near zero field. Only phonon drag, not diffusion, has been seen in CF electron gases, and so discussion of these will be deferred to the next section. In contrast, both diffusion and phonon drag have been seen in CF hole gases.

In the first paper dealing with hole gas CFs at $\nu = \frac{1}{2}$ and $\frac{3}{2}$, Ying et al.[37] showed that the crossover from drag to diffusion occurred in the region of 100 mK. Their results are reproduced in Fig. 5. The authors analysed their diffusion data using Eq. (8) (with

$\beta = 0$). They compared $S^d_{xx}(\frac{1}{2})/S^d_{xx}(\frac{3}{2})$ and claimed good agreement with the CF model. The details of the arguments will not be reproduced here. A later theoretical paper[38] proposed alternative explanations, though still within the CF approach. Nevertheless, the experiments supported the idea that the behaviour of CFs at these filling factors is very similar to that observed for 2D systems at zero field. Khveshchenko[39] has predicted corrections to the diffusion thermopower logarithmic in temperature, but the experimental data are not precise enough to reveal these.

In a subsequent paper Bayot et al.[9] focussed on the quantum oscillations of CFs and made a rough separation into diffusion and phonon drag parts. They analysed the former in terms of the disorder free result $S^d_{xx} = -\mathcal{S}/ne$, which, at higher temperatures, corresponds to Eq. (15). At lower temperatures and arbitrary kT/Γ (but retaining the assumptions $\hbar\omega_c \gg kT$, Γ) an extension due to Zawadzki and Lassnig[40] was used. A later theoretical paper by Karavolas et al.[41] gave a more detailed analysis based on Eq. (6) and incorporating a model density of states and scattering, and obtained reasonably good agreement with the experimental data. Other data obtained by Crump et al.[42] on a similar system were also analysed by Karavolas et al.[41], again with reasonable agreement. In both cases the data and analysis spanned a wide range of $kT/\hbar\omega_c$. It would be interesting to compare data on both $\tilde{\rho}_{xx}$ and $\tilde{S}_{xx}$ to determine if the expected phase shift at low fields also occurs in these systems.

Finally, we mention the experiments of Bayot et al.[43] in the fractional quantum Hall regime but which are not related to CFs. The thermopower of a 2D hole gas was examined in the re-entrant insulating phase around $\nu = \frac{1}{3}$. ρ_{xx} tends to zero at $\nu = \frac{1}{3}$, but for ν somewhat above or below this value ρ_{xx} grows exponentially with $1/T$, i.e., $\rho_{xx} \sim \exp(\varepsilon_A/k_BT)$ where ε_A is an activation energy. The reason why the 2D gas becomes insulating in this region is still controversial, e.g. see Ref(44) for the electron case, but the formation of a Wigner crystal is one possibility. Bayot et al. found that the thermopower, which they identified with S^d_{xx}, begins to diverge at low temperatures. Intrinsic semiconductors at zero field also show a diverging thermopower according to $S = C(k_B/e)(\varepsilon_g/2T + A)$ where A and C are constants of order unity and ε_g is the energy gap; an example is given by Tauc[3] for InSb. When there is only one type of carrier, the prefactor C is unity and the result is Eq. (10). Burns and Chaikin[45] have used this equation to argue for an energy gap in 2D metal films. Bayot et al. suggested that their data could be analysed in the same way to yield an energy gap E_g in the insulating state. As we have mentioned earlier, semi-classically Eq. (10) reduces to -$\mathcal{S}/ne$ as $\beta \to \infty$. Because of this identification with entropy, the use of this result by Bayot et al. is intuitively reasonable, but we should be cautious in assuming that it necessarily holds at arbitrary magnetic fields in the FQH insulating region. The analysis appeared to yield the same energy gap from either ρ_{xx} or S^d_{xx}, though the thermopower data do not provide a very convincing $1/T$ behaviour.

B. Phonon Drag Results

Many recent developments in this area have been concerned with experimental investigations of the validity of Eq. (18), though there are some other notable results which will be discussed later. The evidence concerning the prediction embodied in Eq. (18) that $\overleftrightarrow{S}^g$ is diagonal will first be discussed, and the relationship between μ_{ep} and S^g will be left until later. This part of the discussion is limited to the field region where quantum oscillations are small or negligible.

It had been noticed on many occasions in the past that S^g_{xx} appeared to be independent of magnetic field but the results of Fletcher et al.[30] on a GaAs/Ga$_{1-x}$Al$_x$As quantum well (also mentioned in the previous section with respect to diffusion) were the first to systematically examine the field dependence of the smooth parts of both S_{xx} and S_{yx}. The field range up to about $\omega_c\tau_t \approx 1$ at temperatures $1 < T < 200\,\mathrm{K}$ was examined. Phonon drag was dominant over most of the temperature range and in these regions S^g_{xx} was accurately independent of B in agreement with predictions. However, S^g_{yx} was not zero, though it was rather small. Later work on a Si-MOSFET[35] also showed similar features, in particular the appearance of what appeared to be phonon drag in S_{yx} (see Fig.3). These observations contrast with the results on 3D HgSe:Fe by Tieke et al.[7] mentioned in the last section where S^g_{xx} also appeared to be constant, but S^g_{yx} was accurately zero.

Eq. (18) was obtained with an isotropic model for both the electrons and phonons. Very recently, Butcher and Tsaousidou[23] have shown that when anisotropy in both the phonon and electron systems is present, then departures occur from Eq. (16) between ϵ^g_{ij} and σ_{ij}. This results in the two terms in Eq. (3) no longer precisely cancelling so that a non-vanishing S^g_{yx} is predicted. This may be sufficient to explain the experimental results mentioned above but it is not clear why HgSe:Fe should obey the predictions of the isotropic model so accurately. The explanation also implies a field dependence of S^g_{xx} from Eq. (4) but this has not yet been evaluated to determine if it is consistent with the measurements.

In connection with this, it has been known since the first measurements that S^g_{yx} also showed features in the quantum Hall regime that were not understood. The early work is fully discussed in Ref. 1. The main feature of interest here is that S^g_{yx} shows an oscillatory behaviour which changes sign and passes through zero when S^g_{xx} is at a maximum. This behaviour mimics that expected for S^d_{yx} (e.g. see Ref. 21) but the magnitude, temperature dependence and other features all indicate that it must be a phonon drag effect.[1] Recently Tieke et al.[46] showed that all previously published oscillatory data on S^g_{yx} are consistent with the simple empirical result

$$S^g_{yx} = \gamma_s B(\partial S^g_{xx}/\partial B) \tag{19}$$

where γ_s is a numerical constant of about 0.05. This is analogous to the relation

$$\rho_{xx} = \gamma_r B(\partial \rho_{xy}/\partial B) \tag{20}$$

that had already been discovered to hold for the resistivities and, further, it was found that $\gamma_r \approx \gamma_s$. An example is shown in Fig. 6 for data in the integer quantum Hall region, and data taken in the fractional quantum Hall region are equally well reproduced by this empirical equation.[46] A similar relation has been found to hold in experiments probing the interaction of surface acoustic waves with a 2DEG.[47] So far, the most convincing explanation for these empirical relations was given by Simon and Cooper.[48] They argued that it is results from the effect of inhomogeneities in the 2DEG. The theory also requires $|\rho_{yx}/\rho_{xx}| \gg 1$ or $|S_{xx}/S_{yx}| \gg 1$ so it seems unlikely that this mechanism is responsible for the low field value of S^g_{yx} not being zero.

On a different tack, Cao et al.[49] examined the field dependence of S_{ij} for two samples of InAs/GaSb heterostructures. The novelty of these samples was that they had a 2D electron gas on the InAs side of the interface, and a hole gas of similar density on the GaSb side. The simplest approach to understand transport with two groups of carriers conducting in parallel is to assume that interband scattering is absent, and that their contributions to the transport tensor coefficients in Eq. (1) are simply additive; this is the basis of most results for the magnetoresistivity of two groups of carriers quoted in standard texts. However, if one attempts to extend this approach to S_{ij} for two groups of carriers in a magnetic field, and to write down the results in terms of the S_{ij} of the individual groups, the expressions are completely unwieldy. A much better way of comparing experiment with theory is to evaluate the relevant ϵ_{ij} from the experiments and compare these with the theoretical results. This was the approach taken by Cao et al.[49]

The data were dominated by phonon drag, but Eq. (16) for ϵ^g_{ij} was not available at that time. However, the results of Zianni et al.[15], as described in Section IV B, predicted S^g_{xx} to be constant, say S^g_0, and $S^g_{yx} = 0$. These were used with free electron results for $\overleftrightarrow{\sigma}$ to obtain $\overleftrightarrow{\epsilon}^g = \overleftrightarrow{\sigma}\overleftrightarrow{S}^g$; the resulting equations for ϵ^g_{ij} have the same field dependence as those in Eq. (16), i.e.

$$\epsilon^g_{xx} = \frac{\sigma_0}{1+\beta^2} S^g_0 \tag{21}$$

$$\epsilon^g_{yx} = \frac{\beta\sigma_0}{1+\beta^2} S^g_0 \tag{22}$$

where σ_0 is the zero field conductivity. It was then assumed that two similar contributions were to be added for each of the ϵ^g_{ij}, one from the electrons and one from the holes. In principle one should also include diffusion terms and this was attempted, but these were discarded for most of the fits. The resulting equations for ϵ^g_{ij} were fit to the experimental data to obtain the optimum parameters. It turns out that there are far fewer free parameters than one would have initially expected, yet the resulting fits are very good over a wide field range. Fig. 7 shows an example for one of the samples. It should be noted that ϵ^g_{xx} is opposite in sign for electrons and holes, but ϵ^g_{yx} has the same sign. Physically this is because the combined action of a temperature gradient and magnetic field sweeps opposite-sign carriers to opposite sides of the sample.

In view of the fact that $S^g_{yx} = 0$ was not found to be experimentally valid in other 2D systems, one might question if its use in the way described above is an accurate procedure in this case. There are two observations that can be made about this. First, note that the magnitudes of S_{yx} and S_{xx} are very similar for this system so that the effect of inhomogeneities, as mentioned above, is unlikely to be important. Second, it is important to realize that in Eq. (3) the two terms $\rho_{xx}\epsilon_{yx}$ and $\rho_{yx}\epsilon_{yy}$ are each very large and cancel to give $S^g_{yx} = 0$ only if $\epsilon^g_{ij} \propto \sigma_{ij}$ holds precisely. To explain the observed non-zero value of S^g_{yx} in Ref. 30 requires a rather small imbalance in the two terms, as shown quantitatively by Butcher and Tsaousidou.[23] One can also take the pragmatic view that, whatever the cause of the discrepancies observed for S^g_{yx} in other situations, the experiments of Cao et al.[49] confirm the basic validity of the field dependences predicted by Eq. (16).

The new feature in Eq. (18) which was not explicit in the previous theoretical work, namely that the magnitude of S^g_{xx} is directly related to μ_{ep}, will now be examined. As mentioned earlier, piezoelectric e-p coupling dominates e-p scattering in GaAs/$\mathrm{Ga}_{1-x}\mathrm{Al}_x$As 2D samples at low temperatures. In the limit $\langle q \rangle \ll 2k_F$, where $\langle q \rangle$ is the average magnitude of the phonon wave vector and k_F the magnitude of the Fermi wave vector, theory predicts[50] screened piezoelectric scattering to give $\mu_{ep} \sim T^{-5}$. Zero field conductivity data on a high mobility 2DEG at a GaAs/$\mathrm{Ga}_{1-x}\mathrm{Al}_x$As heterojunction were analysed by Stormer et al. to obtain μ_{ep} at low temperatures,[50] the results, reproduced in Fig. 8, being in agreement with expectations in both temperature dependence and magnitude. This was followed by experiments[51] where the same quantity was extracted for CFs, say μ_{cfp}, at $\nu = \frac{1}{2}$. The result was $\mu_{cfp} \sim T^{-3}$, the data also being shown in the figure.

Very recent experiments by Tieke et al.[8] used phonon drag and Eq. (18) to obtain the same two quantities, μ_{ep} for electrons and μ_{cfp} for CFs at $\nu = \frac{1}{2}$, also for GaAs/Ga$_{1-x}$Al$_x$As samples. With electrons at zero B, a T^4 dependence was expected and observed for S^g_{xx} at low temperatures (there was no evidence of a diffusion contribution); this was converted to μ_{ep} using Eq. (18) and good agreement was found with the earlier data from conductivity,[50] and also with theoretical predictions. Fig. 8 also shows these results.

Note that, as sample mobility is decreased, it rapidly becomes impossible to extract μ_{ep} from conductivity measurements. This arises not only because e-p scattering contributes a diminishing fraction to the total resistivity, but also because the dominant e-i scattering has residual temperature dependences due to the variation of screening with temperature and possibly other effects such as weak localization. On the other hand, S^g is not sensitive to e-i scattering (or weak localization as will be discussed below) and continues to be useful to arbitrarily low mobility. The other experimental method which is often used to give information on the e-p interaction is the measurement of the energy loss rate of the carriers. This method is complementary in that it measures the energy relaxation rate, whereas conductivity and thermopower measure the momentum relaxation rate.

Phonon drag also dominates S_{xx} for 2DEGs at high magnetic fields. The first results by Zeitler et al.[52] on S^g_{xx} were completely consistent with the CF model, though this was not appreciated at the time. These authors noted that 'the $\frac{1}{3}$ filling factor acts as a boundary between two different states', which we now recognize as the CF states at $\nu = \frac{1}{2}$ and $\frac{1}{4}$. Tieke et al.,[53–55,8] confirmed this and showed that the behaviour of S^g_{xx} for CFs at $\nu = \frac{1}{2}$ and $\frac{3}{2}$ was very similar to that of electrons at $B = 0$, S^g_0, as the model for CFs would generally predict. Surprisingly, the measured thermopowers[55,8] for filling factors which have the same denominator appear to be identical, i.e., $S^g(\frac{1}{2}) = S^g(\frac{3}{2})$ and $S^g(\frac{1}{4}) = S^g(\frac{3}{4})$, but the reason for this not understood. Concentrating on the results at $\nu = \frac{1}{2}$, it was found that $S^g(\frac{1}{2}) \sim T^{3.5\pm0.5}$, compared with $S^g_0 \sim T^{4.0\pm0.5}$, though the magnitude for CFs was roughly 60 times larger. CF hole gases do not seem to show this large increase in magnitude of S^g_{xx} compared to hole gases at zero field,[37,42] but why this should be so is also unknown. Using

Eq. (18), μ_{cfp} was calculated from S^g_{xx} for the CF electron gases,[8] but in this case there was no agreement with the data from conductivity.[51] From thermopower $\mu_{cfp} \sim T^{-4.5}$, whereas conductivity[51] gave $\mu_{cfp} \sim T^{-3}$. All the μ_{cfp} data are collected in Fig. 8.

There seem to be two possible reasons for the discrepancy between the thermopower and conductivity results for μ_{cfp}. As noted above, e-p scattering has only a small effect on the conductivity, and various corrections for other temperature dependent effects must be made before μ_{cfp} can be extracted. Thus it is possible that the μ_{cfp} cannot be reliably obtained in this manner. On the other hand, provided CFs obey the Boltzmann equation, then the results obtained from thermopower should be accurate.

Another possibility arises from the work of Khveshchenko and Reizer[56] who find that piezoelectric e-p coupling is modified in the dirty limit $\langle q \rangle l_e < 1$, where l_e is the impurity limited mean free path of the carriers. A consistent explanation of the temperature dependence of both the conductivity and the thermopower can be obtained if one assumes that the conductivity sample was in the clean limit and the thermopower sample was in the dirty limit. However, if one uses $\langle q \rangle \sim kT/\hbar v_s$, both samples should be in the clean limit over the whole temperature range studied. In passing, it should be noted that it is implicit in these theoretical results[56] that Eq. (18) holds for CFs in the clean limit, but it does not hold in the dirty limit.

In principle, the thermopower data on hole CFs at $\nu = \frac{1}{2}$ of Ying et al.[37] and Crump et al.[42] could be analysed in a similar manner, but this has not been done. The latter data were used to provide evidence that CFs have a well defined Fermi surface (or circle). Theory predicts a peak in S^g_0/T^3 when $\langle q \rangle \sim 2k_F$ (see Ref. 1) and this is seen in the CF data, just as it was in the electron data at zero field.

A recent paper has reported on the thermopower of double p-type quantum wells.[57] There is expected to be a strongly correlated state for the two wells when $\nu = 1$, ($\frac{1}{2}$ in each layer), and the thermopower and resistivity measurements are consistent with this. In addition both the thermopower[57] and resistivity[58] show hysteresis behaviour when $\nu = 2$, (1 in each layer). According to theory[59] a rich variety of magnetic phases might be possible at this particular filling factor, and the experiments may be sensitive to transitions between these phases, or perhaps to domain structure within one of the states.

Finally, some new experimental data concerning thermopower and weak localization (WL) will be outlined. WL is a quantum correction to the conductivity which arises because the phase of the electronic wavefunction is not randomized during elastic collisions. Interference effects between different propagating paths for the electrons cause a decrease in the diffusivity, and hence conductivity. The conductivity can be restored by the application of a weak magnetic field (and also by increasing the temperature, but this is not pursued here). The effects of WL on $\overleftrightarrow{\sigma}$ have been thoroughly explored, and the question of interest here is the effect of WL on $\overleftrightarrow{\epsilon}$, or equivalently on $\overleftrightarrow{S}$. Ref. 1 drew attention to this problem and reviewed the situation at that time. It is necessary to briefly outline the previous work so that the new work can be placed in perspective. All available data are for 2D systems.

In a series of papers on a low mobility Si-on-sapphire MOSFET, Syme et al.[60] published results on the magnetic field dependence of both σ_{xx} and S_{xx} in the region where σ_{xx} shows easily observable effects due to WL. If one writes the relatively small changes in σ_{xx}, etc, due to the effect of B on WL as $\Delta\sigma_{xx}$, then Eq. (4) gives

$$\frac{\Delta S_{xx}}{S_{xx}} = \frac{\Delta\epsilon_{xx}}{\epsilon_{xx}} - \frac{\Delta\sigma_{xx}}{\sigma_{xx}} . \tag{23}$$

WL is a low field effect such that $\omega_c\tau_t \ll 1$ and advantage has been taken of this by ignoring the term $\rho_{yx}\epsilon_{xy} \propto (\omega_c\tau_t)^2$, and also in putting $\rho_{xx} = 1/\sigma_{xx}$. It is expected that there will be a correction to S^d_{xx} due to WL,[1] but experimental data were obtained at temperatures where S^g_{xx} was dominant and so it was assumed that the observed field dependence was mainly due to S^g_{xx}. The main result of the work was that $\Delta S^g_{xx}/S^g_{xx} \approx -\Delta\sigma_{xx}/\sigma_{xx}$ so that $\Delta\epsilon^g_{xx}/\epsilon^g_{xx}$ was consistent with zero.

As pointed out by Miele et al.,[24] this result is not what one would expect. WL can be considered to be a correction to τ_{ei} and Eq. (16) gives $\epsilon^g_{xx}(B) \sim \tau_{ei}(B)$ (at low field $\sigma_{xx}(B) \propto \tau_{ei}(B)$). Hershfield and Ambegaokar[61] have, in fact, shown that WL can be included in the semiclassical Boltzmann equation by a suitable modification to e-i scattering. Because of these considerations, Miele et al.[24] undertook a new experimental investigation of the problem, this time using two low mobility δ-doped GaAs/Ga$_{1-x}$Al$_x$As quantum wells. Low mobility samples are required for these measurements so that WL conductivity corrections are an appreciable fraction of the total. Again phonon drag was completely dominant. Fig. 9 gives an example of results from this new work. The upper panel shows data in a perpendicular field. It is seen that $\Delta S_{xx}/S_{xx}$ is now an order of magnitude smaller than $\Delta\sigma_{xx}/\sigma_{xx}$, though it is not zero within experimental error. The other sample (not shown here) has an even smaller variation of $\Delta S_{xx}/S_{xx}$ with B. As with Syme et al.,[60] it was argued that this result essentially reflects $\Delta S^g_{xx}/S^g_{xx}$. Thus one concludes that $\Delta\epsilon^g_{xx}/\epsilon^g_{xx} \approx \Delta\sigma_{xx}/\sigma_{xx}$ which is completely contrary to the earlier results but is now in keeping with the reasoning outlined above. Possible reasons why $\Delta S_{xx}/S_{xx}$ is not exactly zero have been discussed in the original paper.[24] For completeness, the lower panel of Fig. 9 shows data taken with a parallel magnetic field, where no changes are expected (at least at low fields), nor seen, in the measured quantities.

It might seem surprising that there was no work on this topic before the series of experiments by Syme et al.[60] There has been a substantial effort involving thermopower and localization in 3D and in thin metal films, some of which was reviewed by Gallagher and Butcher.[1] However, phonon drag was not observed (or not recognized) in this work and the analyses were made solely in terms of diffusion. The reason for the absence of drag is that the low mobility required to observe localization effects is achieved by alloying. In 3D this same alloying also causes a strong reduction of Λ and hence phonon drag because of scattering of phonons by the disordered lattice. In this regard it has recently been suggested that inelastic impurity scattering is the dominant phonon scattering mechanism.[62] It is also found that the ratio of drag to diffusion is much smaller in bulk metals than it is in 2D.

In contrast, with 2D heterojunctions, quantum wells, etc., low mobilities are produced by inserting impurities locally into the 2DEG. These impurities appear to have no observable effect on the substrate phonons which are responsible for phonon drag. Thin metallic films seem to be an intermediate case, but if they are grown on disordered substrates, e.g. glass, then drag would be expected to be very small.

This leads to the interesting conclusion that phonon drag can be investigated in 2D even in the region of strong localization. Thus it may be possible to gain information on the e-p interaction as the system goes from weak to strong localization. The only previous experimental work in this area was by Gallagher et al.[5] on a Si-MOSFET and is reviewed in Ref. 1. These are interesting and important results but their significance with regard to the above comments is as yet unclear. A closely related problem is the effect of strong disorder on the e-p interaction which has recently been the focus of work by Khveshchenko and Reizer[56] (briefly mentioned above with regard to the thermopower of CFs), and Chow et al.[63] on phonon emission by hot electrons.

VI. SUMMARY

It has been shown that a consistent description of most of the magnetothermoelectric properties of 2D and 3D degenerate semiconductors can be given in terms of physical models that are intuitively appealing and which can be supported by detailed calculation. There have been several areas in which substantial progress has been made since the major review by Gallagher and Butcher.[1]

At low fields, both the oscillatory and non-oscillatory contributions to the diffusion thermopower of 2DEGs have now been unambiguously observed and are found to be well described by theory. This is true for both tensor components of the thermopower. The Nernst-Ettingshausen coefficient S_{yx} of HgSe:Fe, the only 3D system so far investigated, can be fully described with only diffusion effects, thus validating the basic model used for diffusion and also confirming that phonon drag is zero for this coefficient. The situation with S_{xx} for the same system is not yet clear and further experiments are required to distinguish two competing theories concerning the mechanism responsible for diffusion oscillations.

Progress has also been made with phonon drag. The theoretical framework of 2D drag established by Butcher's group has provided an understanding of the phenomena since the earliest observations. Recently this has been shown to be equivalent to an interpretation in terms of the electron-phonon momentum relaxation time. This connection provides a simple, but powerful, intuitive model which gives insight into many experimental results. It has been particularly useful in relating phonon drag in S_{xx} to the phonon-limited electron mobility.

Much of the recent work on drag has involved the smooth background at low fields. The basic model predicts that the drag component in S_{xx} is independent of field, which is in agreement with experiment. Some new experimental results indicate that weak localization has either no effect, or only a small effect, on phonon drag in S_{xx}. This is contrary to earlier observations but is completely consistent with the model. A detailed description of this using diagrammatic techniques is still required.

The problem of phonon drag in S_{yx} has proved to be more complex. The prediction of the basic model is that drag makes no contribution at all, though this can be viewed as a cancellation between two large, but opposite sign, contributions. As mentioned above, experimental data in a 3D system completely support this prediction. However, drag contributions to S_{yx} have been seen in many experiments on 2D systems. Deviations from isotropy and perfect homogeneity have been shown to produce a resultant drag component, but it remains to be seen whether the current theoretical models can satisfactorily describe all the observed features.

The same models as used for diffusion and phonon drag thermopower of electrons and holes have also provided a consistent explanation of the thermopower of composite fermions, though many details remain to be understood.

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Note added in proof: Since this review was written Tsaousidou, Butcher and Kubakaddi (submitted for publication) have shown that a calculation based on the Boltzmann equation, with the CF-phonon piezoelctric coupling given in Ref [55], gives excellent agreement with the experimental CF phonon drag thermopower of Ref [8].

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FIGURES

FIG. 1. The resistances of two thermometers R_a and R_b, used to measure the temperature and temperature gradient, as a function of temperature. $R_a(T_2)$ and $R_b(T_1)$ are the resistances with a temperature gradient present.

FIG. 2. Panel (a) shows experimental data on S_{yx} for HgSe:Fe as a function of B at various temperatures. Panel (b) shows calculations of S^d_{yx} using data on the oscillatory resistivity $\tilde{\rho}_{xx}$ and the parameter p as input. These data cover the high temperature range. Other data covering low temperatures are available in the original paper and show much more harmonic content. Taken from Ref. 7

FIG. 3. The upper panel shows experimental data on S_{yx} at various temperatures for a Si-MOSFET. The lower panel shows calculated curves of S^d_{yx} using experimental data on the oscillatory resistivity ρ_{ij} and the parameter p as input. The smooth lines through the experimental data are the same as those in the lower panel, but with the addition of an extra term proportional to B. In both experiment and theory all the data pass through (0,0), so the data are shifted vertically by multiples of 15 μV/K for clarity. Taken from Ref. 35.

FIG. 4. The upper panel shows experimental data on S_{xx} at various temperatures for the same Si-MOSFET as used in Fig. 3. The lower panel shows calculated curves of S^d_{xx} using the same input as with S_{yx}. The smooth lines through the experimental data are the same as those in the lower panel. In both experiment and theory, the data are shifted vertically for clarity. Taken from Ref. 35.

FIG. 5. The thermopower S_{xx} for a 2D hole gas at a GaAs/$Ga_{1-x}Al_x$As heterojunction at zero field and at filling factors of $\nu = \frac{1}{2}$ and $\frac{3}{2}$. The data in the lower temperature region vary $\sim T$ and are identified with diffusion. The more rapid rise $\sim T^3$ at higher temperatures is taken to be due to phonon drag. Taken from Ref. 37.

FIG. 6. The upper panel shows ρ_{xx} for a 2DEG at a GaAs/Ga$_{1-x}$Al$_x$As heterojunction in the integer quantum Hall region. The lower trace is experimental, and the upper trace is calculated using Eq. (20). Similarly the lower panel shows experimental S_{yx} and the calculated curves are from Eq. (19). The data are believed to be dominated by S^g_{yx}. In both panels the calculated curves are offset from zero for clarity. taken from Ref. 46.

FIG. 7. Examples of ϵ_{xx} (upper panel) and ϵ_{yx} (lower panel) for a InAs/GaSb heterojunction which contains both hole and electron 2D gases. In both panels the symbols are experimental data and the lines are the best fits assuming phonon drag ϵ^g_{ij} is dominant. The dashed and dotted lines are the contributions to ϵ^g_{ij} calculated for the electrons and holes individually. Taken from Ref. 49.

FIG. 8. The open symbols show the phonon limited mobility μ_{ep} of a 2DEG at a GaAs/Ga$_{1-x}$Al$_x$As heterojunction at zero magnetic field as determined from S^g_{xx}. The dashed line is the calculated result assuming screened piezoelectric scattering is dominant. The upper solid line represents data as deduced from the resistivity, the cross-hatched region being a measure of the experimental scatter. The solid symbols give μ_{cfp} for CFs at $\nu = \frac{1}{2}$ as determined by S^g_{xx}, and the lowest solid line is μ_{cfp} as deduced from the resistivity. Taken from Ref. 8.

FIG. 9. Experimental data on the relative change of S_{xx} (solid symbols) and σ_{xx} (open symbols) as a function of magnetic field for a low mobility 2DEG in a GaAs/Ga$_{1-x}$Al$_x$As quantum well. The upper panel gives data with the field perpendicular to the 2DEG, the lower panel with the field parallel. The solid line through the low field data for σ_{xx} in the upper panel is a fit using standard theoretical results. Taken from Ref. 24