PHYS 414, Introduction to General Relativity 2006

- **Instructor**: Larry Widrow; Office: 308F Stirling Hall; 533-6858; widrow@astro.queensu.ca

  I do not have official office hours for this course. Instead, you are encouraged to drop by my office or, alternatively, arrange a meeting by email. I can also try to answer brief questions by email.

- **Lectures**: Slot 13, Mon 1:30, Wed 12:30, Fri 11:30, in Stirling Hall 412B

- **Required Text:**

  General Relativity by J. Hartle

- **Tutorials**: There are no tutorials scheduled for the course. However, I will be happy to run problem sessions as the course progresses if there is interest.

- **Assignments**: There will be approximately 8 problem sets over the term. I will announce the problems during the Wednesday lecture and also post them on the course website.

- **Webpage**: The website for the course is **www.physics.queensu.ca/~phys414/** and will contain important announcements and problem set assignements and possibly lecture notes (TBA).

- **Midterms** We will have a 1 1/2 hour midterm in week 7

- **Mark Breakdown**

  - Assignments 25%
  - Midterm 25%
  - Final 50%

Lecture 1 Introduction

Einstein's theory of General Relativity

Since 1916 the accepted theory of classical gravity replacing Newtonian gravity

- more importantly, theory of spacetime replacing picture in special relativity

Interplay between understanding of dynamics + spacetime.

Earliest dynamics - Aristotelian

* objects come to rest unless acted on by a force
* preferred frame - rest frame of lab

Newtonian - Galilean

* objects in uniform motion unless acted on by force (learn about friction)
* inertial frames - frames where $\vec{F}=0$ implies $\vec{a}=0$

absolute time / no speed limit/velocity addition

Special Relativity

light moves at constant speed (in vacuum) - no ether

Lorentz trans. replace Galilean trans

no absolute time

General Relativity - space no longer immutable

gravity is geometry

Broad brush intro - GR comes from incompatibility of Newtonian gravity + SR

Newtonian gravity + Galilean relativity

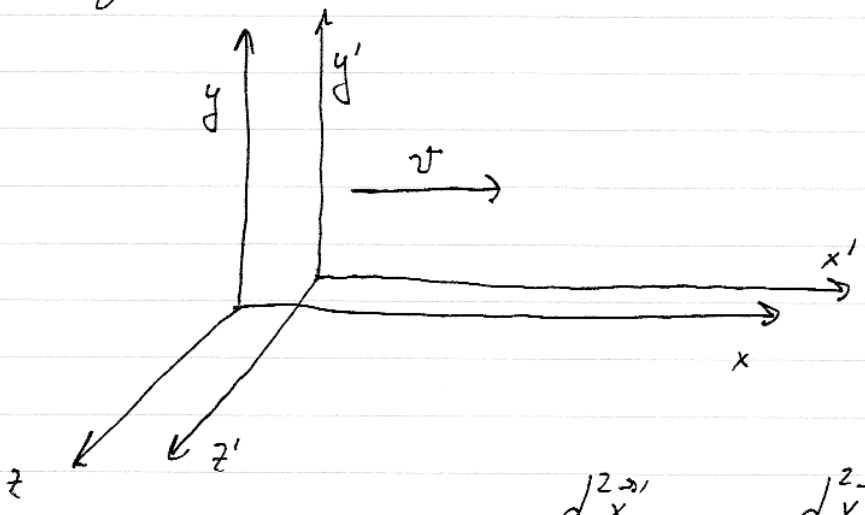
$\vec{F} = m_1 \vec{a}$ How particles move

$\vec{F}_g = -\frac{G m_1 m_2}{r_{12}^2} \hat{r}_{12}$ gravitational force between m_1 & m_2

$= m_1 \vec{g}$ or $\vec{\nabla} \cdot \vec{g} = 4\pi G \rho$

Inertial frame $\vec{F} = 0$ then $\vec{a} = 0$

Experimentally, set up particles where net force is zero + test $\vec{a} = 0$. (pick up subtle effects e.g. rotation of Earth)

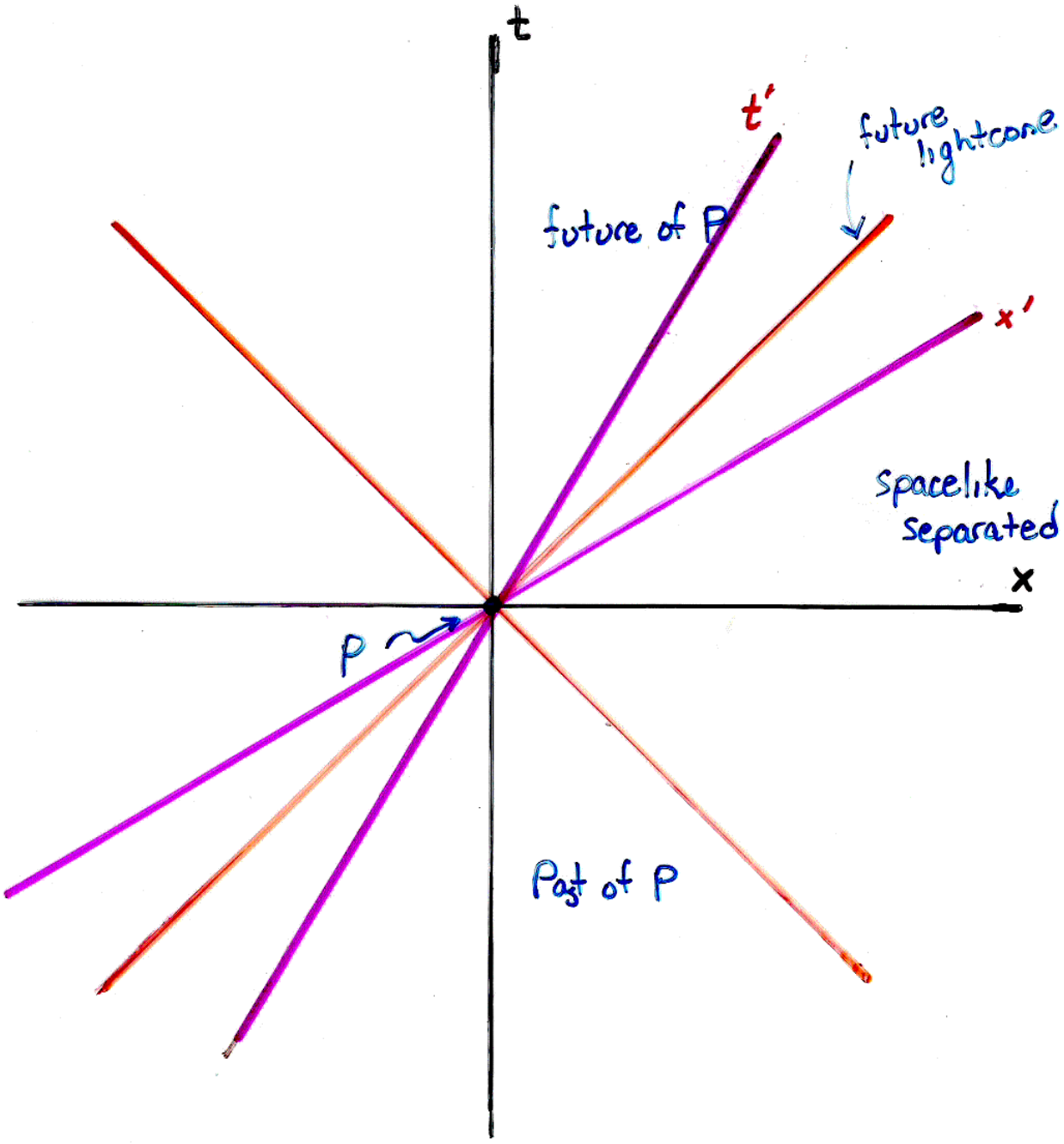


$$x' = x - vt$$

$$t' = t$$

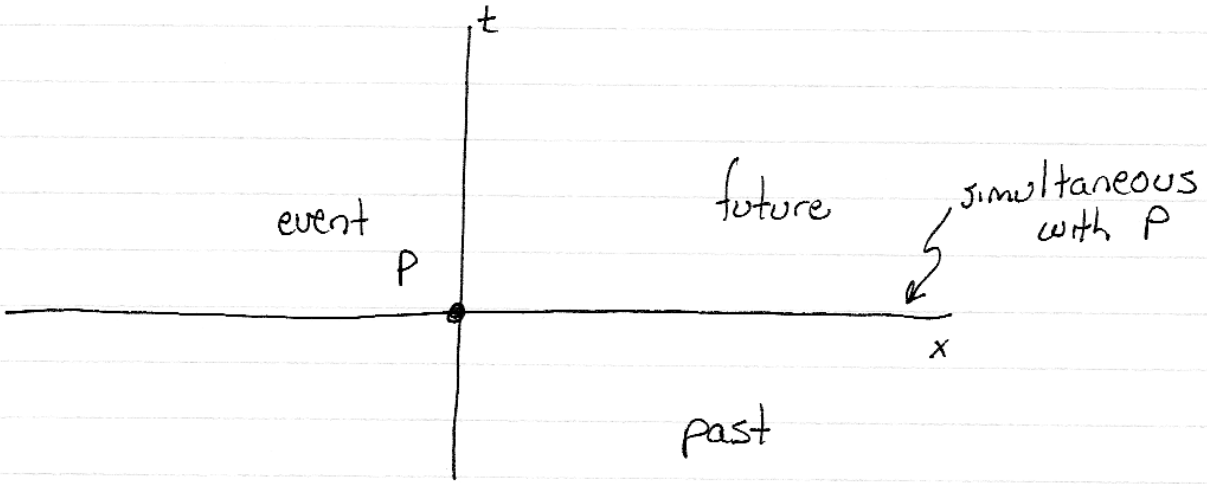
$$y' = y \qquad z' = z$$

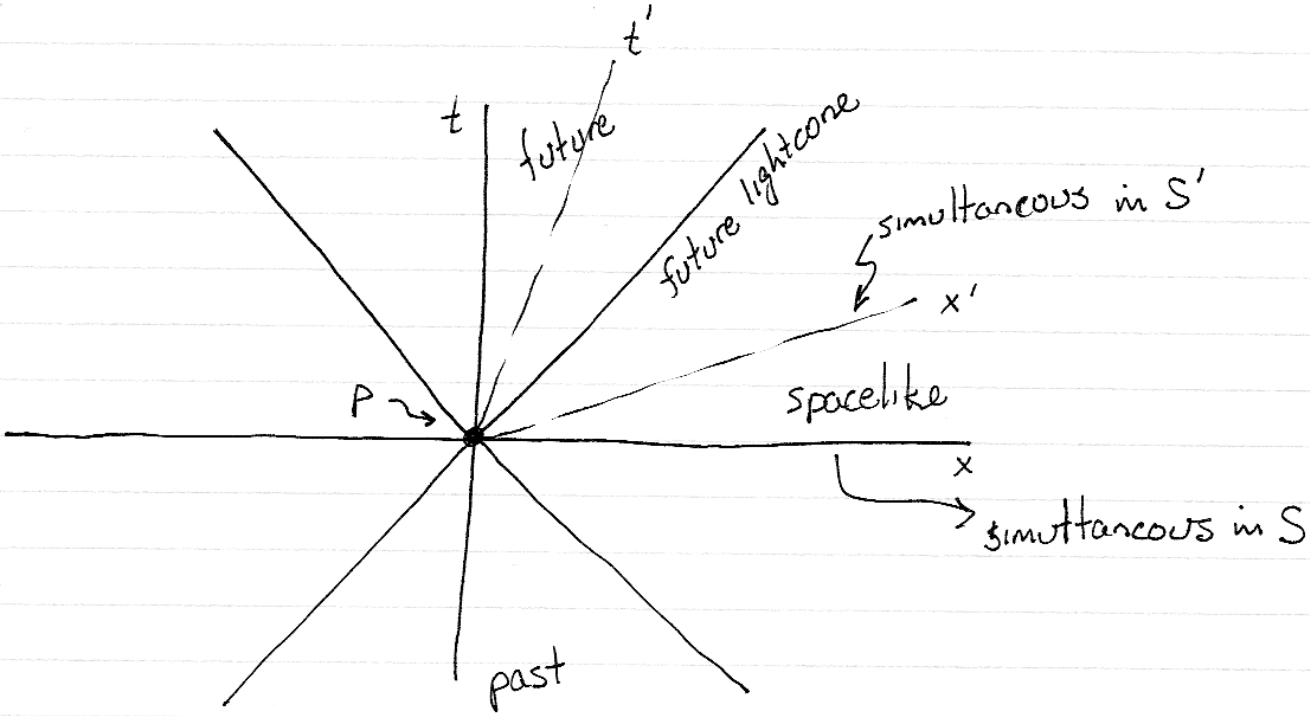
$$\Rightarrow \frac{d^2\vec{x}'}{dt'^2} = \frac{d^2\vec{x}}{dt^2} \quad \text{and} \quad \vec{F} = \vec{F}'$$


t
t'
future lightcone
future of P
x'
spacelike separated
x
P
Past of P

Special Relativity

- Speed of light is the same in all inertial frames
- No signal can propagate faster than c

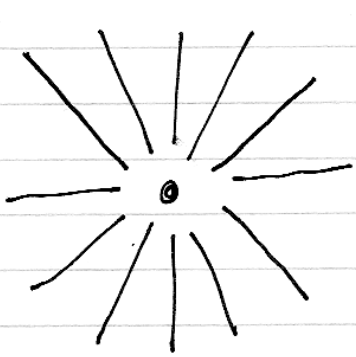


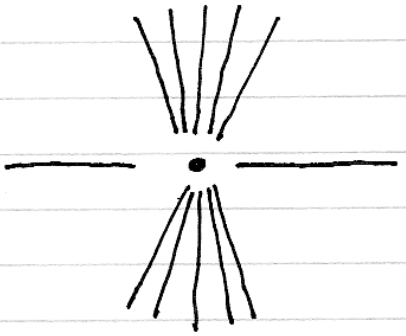


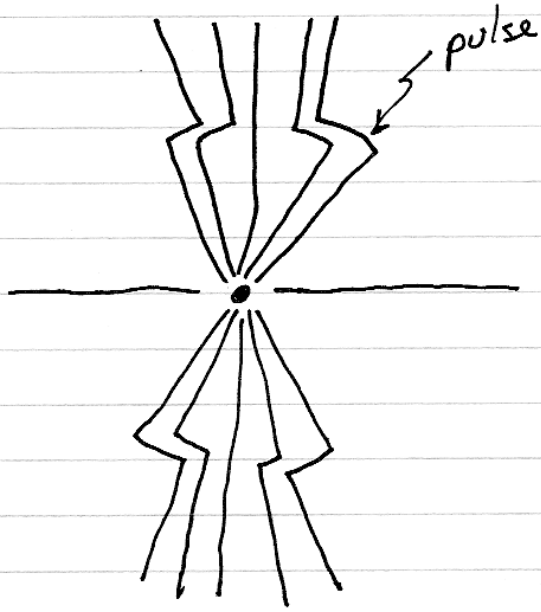
There is a similarity between EM & Newtonian mechanics

$$\vec{\nabla} \cdot \vec{g} = -4\pi G \rho_{matter} \qquad \vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$$

But relativity is built into EM







No such situation exists in Newtonian gravity

- no $\vec{v}$ dependence for $\vec{F}$ ($\vec{F}_g = \vec{F}_g'$ whereas $\vec{F}_{EM} \neq \vec{F}_{EM}'$)

One possibility: $\vec{E} \Leftrightarrow \vec{F}_{\text{Newtonian grav}}$

perhaps there is $\vec{B}_{grav}$

Alternatively consider implications of equivalence principle

To measure $\vec{E}$ and $\vec{B}$

1) use neutral particles to establish inertial frame
2) charge particle initially at rest to measure $\vec{E}$
3) + in motion to get $\vec{B}$

With gravity, no way to measure $\vec{g}$

$$\vec{F} = m\vec{a} = m\vec{g}$$

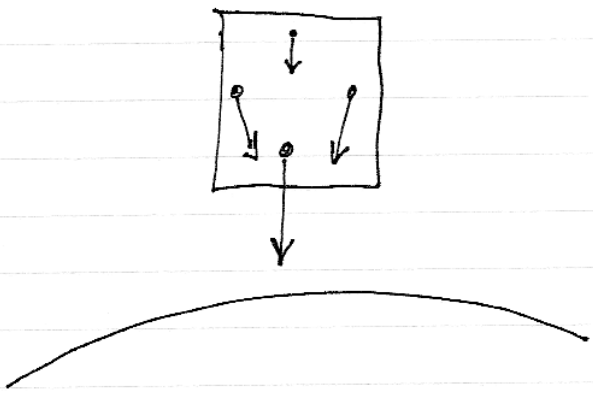
same m

cannot distinguish uniform $\vec{a}$ from uniform $\vec{g}$

all particles follow same path in spacetime

Can we eliminate effects of gravity e.g. in freely falling frame?

No - tidal fields exist



- freely falling particles (particles with no non-grav. forces) follow geodesics (straight-as-possible paths)
- matter curves spacetime - causes geodesics to move together/apart

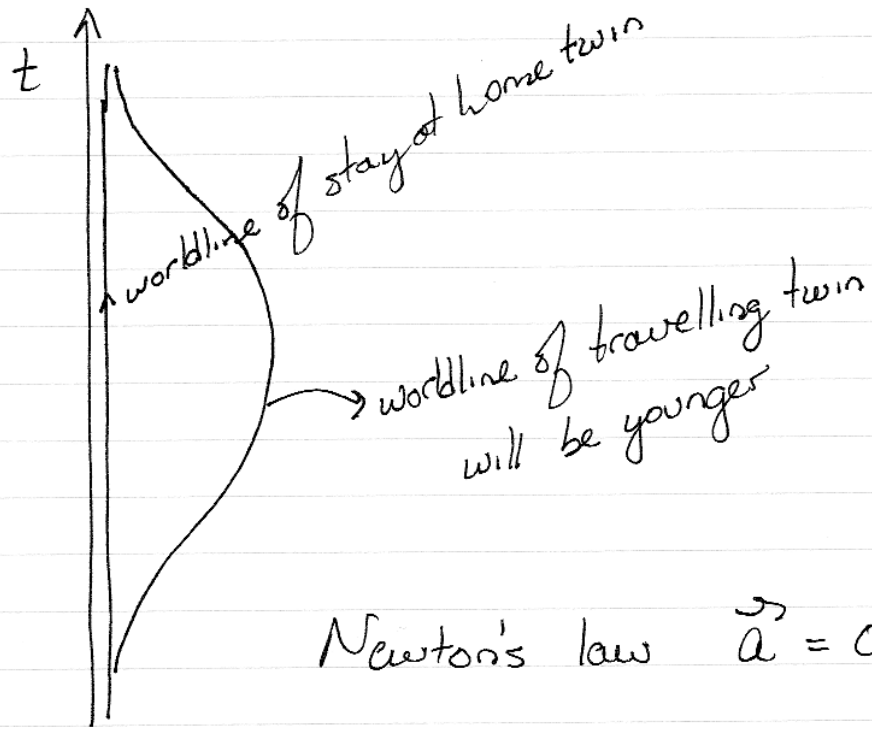
Special relativity - basis for GR - bootstrap method

Go to freely falling frame

formulate laws using SR

add in curvature with principles of covariance of laws under general coordinate transformation

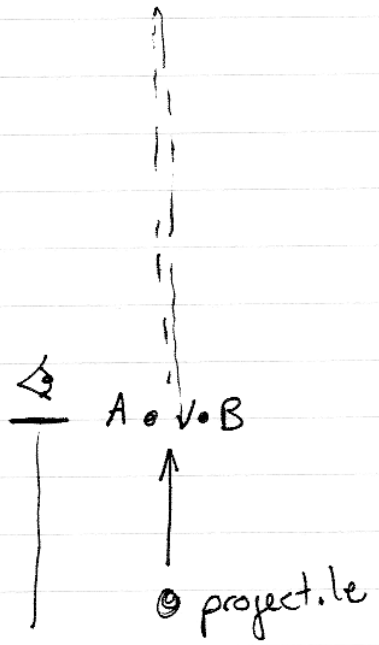
e.g. twin paradox



Newton's law $\vec{a} = 0$ if $\vec{F} = 0$ inertial

$\equiv$ inertial observer between two events is one with max. proper time

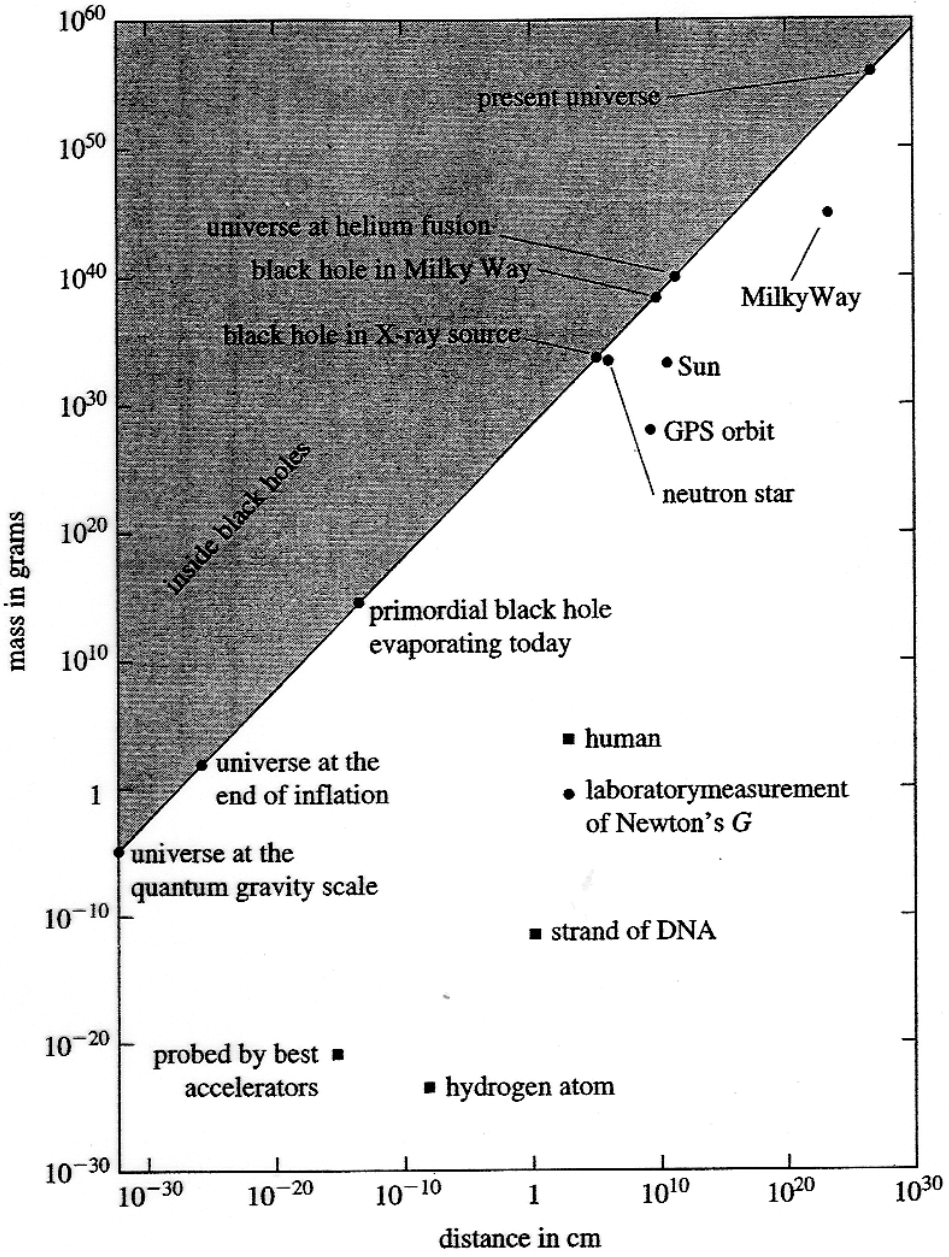
goes over to GR.



events A + B

which is "older"

observer on project.le or on platform?


10^{60}
10^{50}
10^{40}
10^{30}
10^{20}
10^{10}
1
10^{-10}
10^{-20}
10^{-30}
mass in grams
present universe
universe at helium fusion
black hole in Milky Way
black hole in X-ray source
inside black holes
MilkyWay
Sun
GPS orbit
neutron star
primordial black hole
evaporating today
human
universe at the
end of inflation
laboratorymeasurement
of Newton's G
universe at the
quantum gravity scale
strand of DNA
probed by best
accelerators
hydrogen atom
10^{-30}
10^{-20}
10^{-10}
1
10^{10}
10^{20}
10^{30}
distance in cm

Consider spherical mass M, radius R

Test particle in circular orbit

$$\frac{GM}{R^2} = \frac{v^2}{R} \quad \Rightarrow \quad \frac{GM}{R} = v^2$$

In Newtonian gravity, v can take any value (no scale)

In GR, velocity scale is c

$$\frac{v^2}{c^2} = \text{dimensionless ratio} = \frac{GM}{c^2 R}$$

object	GM/c^2R
Earth	10^{-9}
Sun	10^{-6}
Neutron stars	10^{-1}
black holes	1

Why study GR?

- Best currently available description of space + time + gravity
- Testable
- Crowning achievement of the greatest physicist of our time
- Introduces active area of research - classical GR
- basis for cosmology + compact objects
- potential new window to the Universe - gravitational waves
  LIGO LISA
- lensing - strong, weak, micro
- biggest surprise in cosmology - Λ
- jumping off point for string theory