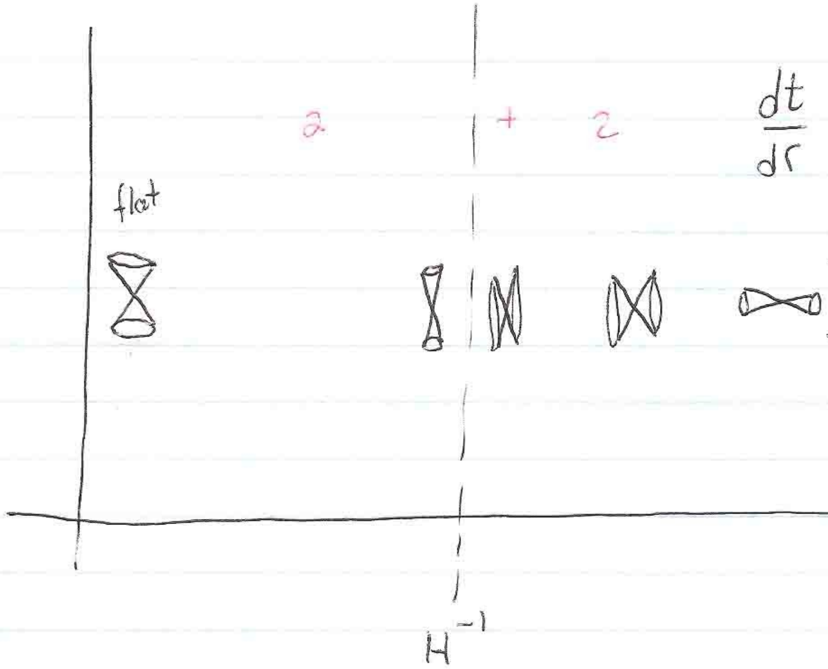
1 a)



$$\frac{dt}{dr} = \pm \left(1 - H^2 r^2\right)^{-1}$$

$\rightarrow \pm\infty$ for $r \rightarrow H^{-1}$

$\rightarrow 0$ for $r \rightarrow \infty$

4

b) i) spherical symmetry so pick $\theta = \pi/2$ plane $d\theta = 0$

2 time-independence so let $t =$ cons. $dt = 0$

ii) 1 $d\rho^2 + \rho^2 d\varphi^2 + dz^2$

let $\rho = \rho(r,\phi)$ $\varphi = \varphi(r,\phi)$ $z = z(r,\phi)$

choose $\varphi = \phi$, $\rho = r$, $z = z(r)$

$$ds^2_{surface} = dr^2 + r^2 d\phi^2 + (z')^2 dr^2$$

1 so require $1 + z'^2 = \dfrac{1}{1 - r^2 H^2}$

$$(z')^2 = \frac{1}{1-r^2H^2} - 1 = \frac{r^2H^2}{1-r^2H^2}$$

$$z' = \frac{rH}{(1-r^2H^2)^{1/2}}$$

$$z = \int \frac{rH\,dr}{(1-r^2H^2)^{1/2}}$$

$$= \frac{1}{H}(1-r^2H^2)^{1/2} \qquad z = \frac{1}{H} \text{ at } r=0$$

this is a sphere =

2a) $\xi^\alpha = (1,0,0,0) \qquad e = -\underset{\sim}{\xi} \cdot \underset{\sim}{u} = (1+A^2r^2)\dfrac{dt}{d\tau}$

3 $\quad \eta = (0,0,0,1) \qquad l = \eta \cdot \underset{\sim}{u} = r^2 \dfrac{d\phi}{d\tau}$

b) $$1 = (1+A^2r^2)\left(\frac{dt}{d\tau}\right)^2 - (1-A^2r^2)^{-1}\left(\frac{dr}{d\tau}\right)^2 - r^2\left(\frac{d\phi}{d\tau}\right)^2$$

3 $$= \frac{e^2}{1+A^2r^2} - \frac{1}{1+A^2r^2}\left(\frac{dr}{d\tau}\right)^2 - \frac{l^2}{r^2}$$

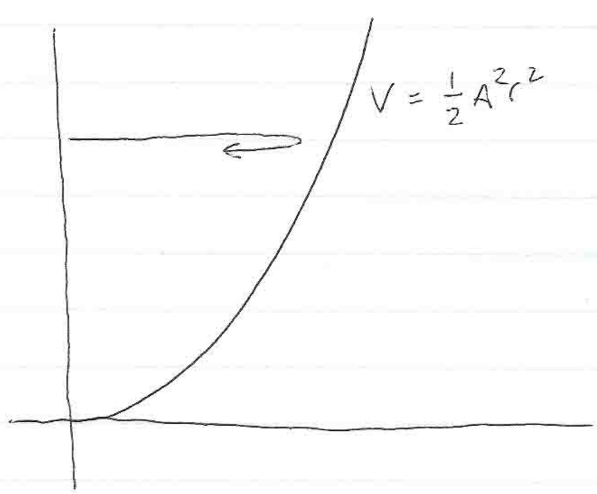
$$\frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 + \frac{1}{2}\frac{l^2}{r^2}(1+A^2r^2) + \frac{1}{2}(1+A^2r^2) = e^2$$

But 2nd term has $\frac{1}{2} l^2 A^2$

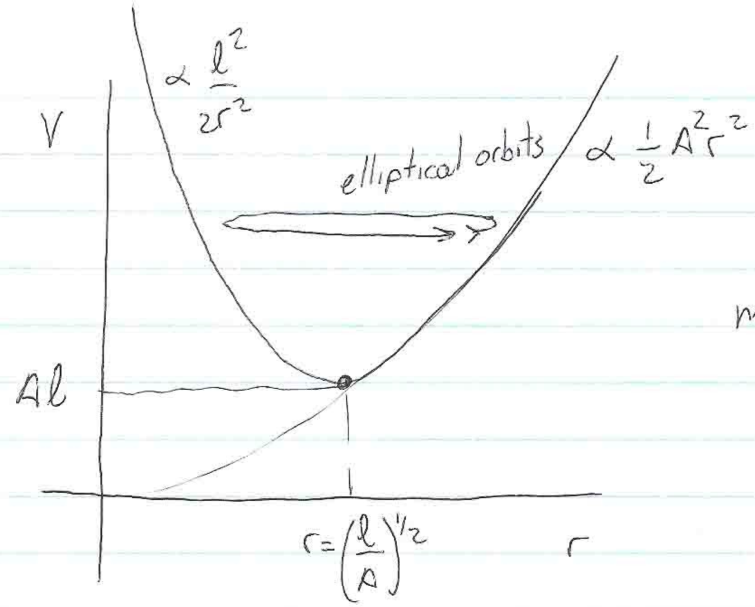
$$\frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 + \frac{1}{2}\left(A^2r^2 + \frac{l^2}{r^2}\right) = \frac{e^2 - 1 - l^2A^2}{2}$$

c) $l = 0$

3



particles orbit
thru center



min at $A^2 r - \frac{l^2}{r^3} = 0$

or $r = \left(\frac{l}{A}\right)^{1/2}$

d) $\frac{d\phi}{d\tau} = \frac{l}{r^2}$ $\quad r = (l/A)^{1/2}$

2 $\quad = A \implies T = \frac{2\pi}{A}$

3 $\frac{d\phi}{dt} = \frac{l}{r^2}\,\frac{1 + A^2r^2}{e} = A\,\frac{1 + Al}{e}$

2

need e: for $\frac{dr}{d\tau} = 0$ + $r = (l/A)^{1/2}$ we have

$$Al = \frac{e^2 - 1 - l^2A^2}{2} \quad \text{or} \quad e^2 = 1 + A^2l^2 + 2Al$$
$$= (1 + Al)^2$$

so $e = 1 + Al \implies \frac{d\phi}{dt} = A \quad T = \frac{2\pi}{A}$

e) $l = 0$ $\frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 + \frac{1}{2}A^2r^2 = \frac{e^2-1}{2}$

initial conditions $\frac{dr}{d\tau} = 0$ for $r = R \Rightarrow \frac{e^2-1}{2} = \frac{1}{2}A^2R^2$

so $\left(\frac{dr}{d\tau}\right)^2 = A^2(R^2-r^2)$

$$\frac{dr}{d\tau} = A(R^2-r^2)^{1/2}$$

3 $\int d\tau = \frac{1}{4}T = \int \frac{dr}{A(R^2-r^2)^{1/2}} = \frac{1}{A}\int_0^1 \frac{du}{(1-u^2)^{1/2}}$

$$= \frac{\pi}{2A} \Rightarrow T = \frac{2\pi}{A}$$

f) $\frac{dr}{dt} = \frac{dr}{d\tau}\frac{d\tau}{dt} = A(R^2-r^2)^{1/2}\left(\frac{1+A^2r^2}{e}\right)$

$$= A(R^2-r^2)^{1/2}\frac{1+A^2r^2}{(1+A^2R^2)^{1/2}}$$

3

$$= A(R^2-r^2)^{1/2}\left(1-\frac{1}{2}A^2R^2+A^2r^2\right)$$

or $$\int dt = 4\int_0^{R} \frac{1}{A} \frac{dr}{(R^2-r^2)^{1/2}} \left(1+\frac{1}{2}A^2R^2 - A^2r^2\right)$$

$$T = \frac{4}{A}\left(\frac{\pi}{2}\left(1+\cancel{\frac{1}{2}A^2R^2}\right) - \cancel{\frac{\pi}{4}A^2R^2}\right)$$

$$= \frac{2\pi}{A}$$

g) Simple harmonic oscillator