1a) $\xi^\alpha = (1,0,0,0) \qquad e = -\underset{\sim}{\xi}\cdot\underset{\sim}{u} = \dfrac{dt}{d\lambda}$

$\eta^\alpha = (0,0,0,1) \qquad l = (1-\alpha)^2 r^2 \dfrac{d\phi}{d\lambda}$

$\mu^\alpha = (0,0,1,0) \qquad p = \underset{\sim}{\mu}\cdot\underset{\sim}{u} = \dfrac{dz}{d\lambda}$

b) $0 = -\left(\dfrac{dt}{d\lambda}\right)^2 + \left(\dfrac{dr}{d\lambda}\right)^2 + \left(\dfrac{dz}{d\lambda}\right)^2 + (1-\alpha)^2 r^2 \left(\dfrac{d\phi}{d\lambda}\right)^2$

$= -e^2 + \left(\dfrac{dr}{d\lambda}\right)^2 + \dfrac{l^2}{(1-\alpha)^2 r^2} \qquad \dfrac{dz}{d\lambda} = 0$

so $\dfrac{dr}{d\lambda} = l\left(\dfrac{e^2}{l^2} - \dfrac{1}{(1-\alpha)^2 r^2}\right)^{1/2}$

$= l\left(\dfrac{1}{b^2} - \dfrac{1}{(1-\alpha)^2 r^2}\right)$

$r_{min} = \dfrac{b}{1-\alpha}$

c) $$\frac{d\phi}{dr} = \frac{d\phi/d\lambda}{dr/d\lambda}$$

$$= \frac{\cancel{\ell}/(1-\alpha)^2 r^2}{\cancel{\ell}\left(\frac{1}{b^2} - \frac{1}{(1-\alpha)^2 r^2}\right)^{1/2}}$$

$$= \frac{1}{(1-\alpha)^2 r^2\left(\frac{1}{b^2} - \frac{1}{(1-\alpha)^2 r^2}\right)^{1/2}}$$

$$= \frac{1}{(1-\alpha) r^2\left(\frac{1}{r_{min}^2} - \frac{1}{r^2}\right)^{1/2}}$$

d) $$\Delta\phi = 2\int_{r_{min}}^{\infty} \frac{dr}{(1-\alpha) r^2\left(\frac{1}{r_{min}^2} - \frac{1}{r^2}\right)^{1/2}}$$

$$r = \frac{r_{min}}{u} \qquad -\frac{dr}{r^2} = \frac{du}{r_{min}} \qquad = \frac{2}{1-\alpha}\int_0^1 \frac{du}{(1-u^2)^{1/2}}$$

$$= \pi/(1-\alpha) \simeq \pi(1+\alpha)$$

e) $C = 2\pi(1-\alpha)R$

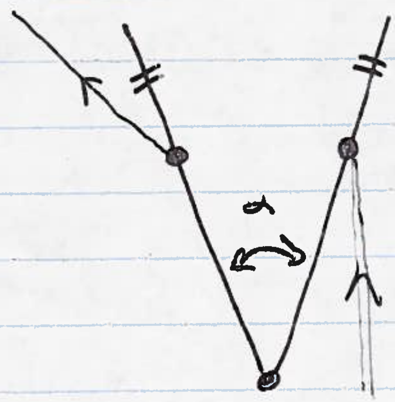
$\Rightarrow$ cone

e.g. $\varphi = (1-\alpha)\phi$

then we have flat space

but $\phi = 0$ + $\phi = 2\pi$ are identified

so $\varphi = 0$ + $\varphi = (1-\alpha)2\pi$ are identified



2a) $u_A^\alpha = (u^t, 0, 0, u^\phi)$

$$u^\phi = \frac{d\phi}{d\tau} = \frac{d\phi}{dt}\frac{dt}{d\tau} = u^t \Omega$$

$$u^\alpha = u^t(1, 0, 0, \Omega)$$

$$\underset{\sim}{u} \cdot \underset{\sim}{u} = -1 = (u^t)^2\left(-1 + \frac{2M}{r} + \Omega^2 r^2\right)$$

so $u_A^t = \left(1 - \frac{2M}{r} - \Omega^2 R^2\right)^{-1/2}$

$$u_A^\alpha = \left(1 - \frac{2M}{R} - \Omega^2 R^2\right)^{-1/2}(1, 0, 0, \Omega)$$

$$u_B^\alpha = \left(1 - \frac{2M}{R} - \Omega^2 R^2\right)^{-1/2}(1, 0, 0, -\Omega)$$

b) $E = -\underset{\sim}{u}_B \cdot \underset{\sim}{p}_B$

or $\gamma = \frac{E}{M_B} = -\underset{\sim}{u}_A \cdot \underset{\sim}{u}_B$

$$= \left(1 - \frac{2M}{R} - \Omega^2 R^2\right)^{-1}\left(1 - \frac{2M}{R} + \Omega^2 R^2\right)$$

$$1 - V^2 = \left(\frac{1 - \frac{2M}{R} - \Omega^2 R^2}{1 - \frac{2M}{R} + \Omega^2 R^2} \right)^2$$

$$V = \left[1 - \left(\quad \right)^2 \right]^{1/2}$$

$$= \frac{\left[\left(1 - \frac{2M}{R} + \Omega^2 R^2\right)^2 - \left(1 - \frac{2M}{R} - \Omega^2 R^2\right)^2 \right]^{1/2}}{1 - \frac{2M}{R} + \Omega^2 R^2}$$

$(a+b)^2 - (a-b)^2$

$= 4ab$

$a = 1 - \frac{2M}{R}$ $\quad b = \Omega^2 R^2$

$$= \frac{2\left(1 - \frac{2M}{R}\right)^{1/2} \Omega R}{1 - \frac{2M}{R} + \Omega^2 R^2}$$

c) $M \to 0$ $\qquad V = \frac{2u}{1 + u^2}$ $\qquad u = \Omega R$

e.g. $\quad x = \gamma(x' + vt') \qquad t = \gamma(t' + vx')$

$$\frac{dx}{dt} = \frac{dx'/dt' + v}{1 + v\,dx'/dt'}$$

let $dx'/dt' = v \Rightarrow \frac{dx}{dt} = \frac{2v}{1 + v^2}$ ✓

d) $$\Omega R = \left(\frac{M}{R}\right)^{1/2} = \frac{1}{6^{1/2}}$$

$$V = \frac{2\left(1-\frac{1}{3}\right)\frac{1}{6^{1/2}}}{1-\frac{1}{3}+\frac{1}{6}} = \frac{2 \times \frac{2}{\cancel{3}}\left(\frac{1}{6}\right)^{1/2}}{\frac{5}{{}_{2}\cancel{6}}}$$

$$= \frac{8}{5} \times \frac{1}{6^{1/2}}$$