1. In the introduction to chapter 9, Hartle makes the claim that in the limit $GM/c^2r \ll 1$, the Schwarzschild geometry, as described by equation 9.1, has the exact same form as Equation 6.20.

   (a) Show that if you use the usual relations between (x, y, z) and (r, θ, ϕ) in equation 6.20, you arrive at a metric with a somewhat different form, even allowing for use of the Taylor expansion in the small parameter GM/c^2r.

   (b) Apply the coordinate transformation

   $$r = r_1 \left(1 + \frac{GM}{2c^2 r_1}\right)^2 \tag{1}$$

   to the Schwarzschild metric and show that the result takes the form

   $$ds^2 = -A(r_1)c^2dt^2 - B(r_1)\left(dr_1^2 + r_1^2 d\theta^2 + r_1^2 \sin^2\theta d\phi^2\right) \tag{2}$$

   Find the functions $A(r_1)$ and $B(r_1)$.

   (c) By a further simple transformation from (r_1, θ, ϕ) to (x, y, z) and with use of Taylor expansions in the small parameter GM/c^2r_1, show that you do indeed have a metric of the form 6.20 with $\Phi = -GM/r_1$

2. Hartle, 9.7