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Spontaneous emission rate enhancements from a single quantum dot embedded in a finite-size, planar photonic-crystal waveguide are investigated. Short waveguide lengths of only 10 to 20 unit cells are found to produce very large Purcell factors associated with a waveguidelike sharp resonance feature in the local density of photon states. Aided by theoretical insight and rigorous computational calculations, we explain the physics behind these remarkable emission enhancements and subsequently propose a “single-photon gun” with on-chip unidirectional collection efficiencies greater than 60% into an output wire waveguide. The advantages over recent proposals for infinitely long photonic-crystal waveguides are highlighted.

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Deterministic single-photon emitters, which seek to exploit the profound consequences of quantum mechanics to achieve unprecedented functionality, are currently one of the most sought after sources in quantum information science [1–5]. Two essential criteria for producing single-photon sources are (i) enhancing the spontaneous emission rates, so that decoherence effects are minimal, and (ii) efficient extraction and thus manipulation of the emitted photons. The first criterion of enhanced spontaneous emission (SE) can be achieved by placing a photon emitter in an environment that increases the local photon density of states (LDOS) through the Purcell effect [6]. Cavity-QED regimes typically rely upon (quasi-) closed cavity systems, whereby one produces a standing-wave electric field in a cavity with a large quality factor $Q$ and a small effective mode volume $V_m$. Therefore, when the cavity mode is resonant with the energy of the photon emitter, the SE can be modified. Less well studied is modified SE in an open cavity system, such as emission in a waveguide or photonic wire, though there have been several interesting predictions; e.g., Kleppner predicted in the early 1980s [10] that photon emission rates can be enhanced using cylindrical-wire waveguides for emission frequencies lying near the mode band edge.

The general concept of single-photon emission into a photonic wire is rich in physics and applications. The photonic wire, cf. a bar of silicon surrounded by air, uses the high-index-contrast to confine light at selected (e.g., telecommunication) wavelengths, using the principle of total internal reflection. By embedding a photon emitter at the center of such a photonic wire [11], fairly large collection efficiencies can be achieved (>50% into a bound propagation mode), though there is no SE enhancement which is necessary for solid-state single-photon emission applications (to overcome decoherence, e.g., caused by electron-phonon interactions). Photonic-crystal (PC) waveguides achieve much richer functionality by trapping light in the photonic band gap typically using air holes in a semiconductor slab, in which a waveguide channel is realized by removing an array of air holes. Consequently, Bragg diffraction can be exploited to achieve both enhanced SE and large collection efficiencies, though this enhanced functionality is considerably more complicated.

Recently, Hughes [12] demonstrated theoretically that large SE factors (>10) can be realized for single quantum dots (QD) embedded in a planar-PC waveguide, and some related results have been reported experimentally using leaky modes [13]. In addition, Lecamp et al. [14] and Manga Rao and Hughes [15] have also demonstrated that broadband (10 THz) single-photon $\beta$-factors of greater than 0.9 are achievable, where the single-photon $\beta$-factor is defined as the probability that an emitted photon will decay into a bound propagation mode [15]. The physical origin of enhanced SE decay in a PC waveguide is an arbitrarily large increase in the LDOS for slow-light modes near the photonic band edge. While such “theoretical proposals” [12,14,15] are attractive and timely with recent experiments, there remains a major problem associated with unavoidable fabrication imperfections that are now known to broaden any divergences in the expected LDOS. Even for so called lossless waveguide modes below the “lightline”, propagation losses scale proportionally with the length of the waveguide and inversely with the mode group velocity [16–19]. Thus, the impact of imperfections for long waveguides will act to broaden the large SE factors and limit the proposed single-photon applications. In addition, it will be very difficult if not impossible to selectively embed only 1 single QD within a long waveguide, at the precise spatial and spectral position.

In an attempt to overcome these problems, in this Letter we study modified SE with a short finite-size PC waveguide of only 10 to 20 unit cells, with realistic boundary conditions similar to those that are always met in an experiment. In terms of efficient photon emission, our
scheme achieves close to the maximum-desired SE factors from single semiconductor QDs, while minimizing the problems of disorder-induced broadening and the technical difficulties of embedding only one QD at a field antinode in a long waveguide structure (typically thousands of unit cells). The maximum-desired SE factor is the enhancement than can be achieved before the onset of strong coupling, which is problematic for single-photon emission. In contrast to the infinite-length waveguide predictions [12,14,15], both the enhanced SE and large extraction efficiencies remain relevant to realistic samples, if they are finite size, even with an integrated coupler added. In addition to being a potentially useful framework for producing single photons, the proposed system gives rise to hitherto unexplored light-matter interactions, which allows producing single photons, the proposed system gives rise to hitherto unexplored light-matter interactions, which allows adding an integrated wire-waveguide coupler [22,23].

Proposal.—In Fig. 1(a) is shown a schematic of a finite-length planar-PC waveguide with a QD placed at the center of the slab. The PC material is expected to exhibit quasibound modes with spectral features similar to those of an infinite system, where light is trapped by the PC in the y direction, bound by total internal reflection in the z direction, and guided by the propagation Bloch modes in x. However, the precise nature of these modes is not known at all due to the termination of the lattice in the propagation direction. As a reference to our detailed analysis given below, in Fig. 1(b) we display the calculated band structure of the fundamental and higher order quasibound modes in an infinite-length planar-PC waveguide, lying within the TE-like band gap (the region between the dark gray areas). Below the air lightline these modes are bound and lossless (ignoring the impact of imperfections), while above the lightline the modes become intrinsically leaky when coupled to the continuum of radiation modes. The LDOS divergences (van Hove singularities) for emitter frequencies lying at the guided-mode band edge are expected, as the group velocity approaches zero. In what follows, we investigate single QDs that are positioned near the maximum of a field antinode in a finite-size PC. Two different structures are considered: (a) a planar-PC waveguide truncated with a definite number of unit cells and surrounded on either side by air, and (b) again a short PC waveguide but surrounded by air only on one end and closed by a defect-free PC on the other end (acting as a reflector). The planar PC is finite in all directions, though the infinite y-direction behavior is quickly realized after only a few unit cells of PC, and so one only needs a small finite number of holes in the y direction; thus most of the finite-size novelty stems from the propagation (x) direction. In addition, we also study the effect of adding an integrated wire-waveguide coupler [22,23].

Theory.—The theory of photon emission in an arbitrary surrounding can be made clear by first calculating the Green function tensor (GFT) [12,24–26]. Although this is in general a formidable task for finite-size structures, it can be determined in the usual way from a dipole solution of Maxwell’s equations, whereby the electric-field GFT, \( \mathbf{G}(\mathbf{r}, \mathbf{r}'; \omega) \), describes the field response at \( \mathbf{r}' \) to an oscillating polarization dipole at \( \mathbf{r} \), as a function of frequency. Once the GFT is known, it is a straightforward task to calculate the SE rates. For example, in the weak coupling regime of interest, the normalized SE factor \( (\equiv F) \) at the QD position \( (\mathbf{r}_d) \) is determined through

\[
F(\mathbf{r}_d, \omega) = \frac{d}{d' \cdot \text{Im} [\mathbf{G}^{\text{PC}}(\mathbf{r}_d, \mathbf{r}_d; \omega)]} \cdot \text{Im} [\mathbf{G}^{\text{hom}}(\omega)] \cdot \frac{d'}{d'},
\]

where \( d \) is a dipole moment of the QD electron-hole pair, and \( \text{Im}[\mathbf{G}^{\text{hom}}] \) corresponds to that of the homogeneous bulk material with refractive index \( n = \sqrt{\varepsilon} \), and \( \mathbf{G}^{\text{PC}} \) is the GFT of the PC. Since the semiconductor QDs of interest are disclike and much smaller than the size of a wavelength, only the in-plane polarization dipoles \( (x, y) \) are relevant and the dipole approximation can be employed. For an exciton energy level resonant with a cavitylike mode \( (\omega_d = \omega_c) \), and located at the field antinode \( (\mathbf{r} = \mathbf{r}_d = \mathbf{r}_\text{ant}) \), the SE factor \( F(\mathbf{r}_\text{ant}, \omega_c) = 6\pi e^3 Q/[(\omega_c \sqrt{\varepsilon})^3 V_m] \).

One can also calculate the “renormalized” response functions, such as the environment-modified relative permittivity of a QD, defined as \( \Delta \varepsilon(\omega) = \varepsilon(\omega)/[1 - V_d \text{Im} [\mathbf{G}^{\text{PC}}(\mathbf{r}_d, \mathbf{r}_d; \omega)] \Delta \varepsilon(\omega)] \), where \( \Delta \varepsilon(\omega) = |d|^2/[2V_d \hbar \varepsilon_0 (\omega_d - \omega - i \gamma_{\text{rad}}/2)] \), \( V_d \) is the volume of the QD, and \( \gamma_{\text{rad}} \) is the nonradiative decay rate. Note that \( \omega_d = \omega_d \) can include a contribution from the Lamb shift (which comes from the real part of the GFT).

Calculations.—We stress that SE calculations for complex finite-size structures are considerably more complicated than that of infinite periodic systems. Consequently, to calculate \( \mathbf{G}^{\text{PC}} \) we have performed a direct numerical solution of the full 3D Maxwell’s equations using finite-difference time-domain (FDTD) techniques [27]. Specifically, we excite a polarization dipole at the desired spatial
In Figs. 2(a) and 2(b) are shown the calculated SE factors for both the y and x-polarization components, although the peak frequencies will be different [cf. solid and dashed curves in Fig. 2(a)]. We remark that similar features in the PC waveguide LDOS have been obtained experimentally [28].

Next, we study the effect of adding five layers of a regular PC to one end of the waveguide to act as a broadband mirror. This is expected to increase the SE while helping the emitted photon to travel only in one propagation direction. In Fig. 3(a) is shown the SE factors in a 10 unit-cell waveguide with air on both sides (solid curve); for a reference, the results for a waveguide with air on both sides are also shown (chain curve). As anticipated, we obtain an increase in $F$ of up to 59 in the case of a PC mirror waveguide with a corresponding $Q = 1000$, in contrast to the air medium case which has a $F = 34$ with a corresponding $Q = 420$. Figure 3(b) displays the corresponding field profiles obtained at the fundamental LDOS peak [shown by a circle in Fig. 3(a)], from which we calculate $V_m \approx 0.144 \mu m^3$.

A key advantage of having a finite-size calculation of the SE factor is that one can investigate the sensitivity of the enhancements to the number of unit cells, which can then be matched in a systematic way to experiments. For example, in Fig. 3(c) is shown the SE factors for a 20 unit-cell waveguide demonstrating that very large SE factors up to 170–175 for an air medium and a PC mirror, achievable with corresponding quality factors $Q = 7600$ and $Q = 6800$, respectively. The smaller adjacent peaks are again due to Fabry-Perot resonances caused by reflection from the PC-air interface. One also notices that in the case of longer waveguides, there is a spectral narrowing and a redshift in the resonances as compared to smaller guides. This behavior is not surprising since the LDOS and peak resonance frequency in longer waveguides must asymptotically converge to the band-edge of ideal infinite-length planar-PC waveguides (where the $F$ is inversely proportional to the group velocity [12]). The field distribution $|E_r|^2$ for the 20 unit-cell PC mirror waveguide case is shown in Fig. 3(d). For single-photon applications one usually desires to maximize the SE while avoiding the regime of strong coupling. As shown in the insets of Figs. 3(a) and 3(c), for a QD dipole strength of $|d| = 50$ D (dashed curve), both the 10 and 20 unit-cell waveguide with a PC mirror give close to the maximum desired SE factor for single-photon emission; in contrast, for a larger dipole.
more than 60% of coupling into the output waveguide (52%). More importantly, with the integrated coupler, the overall SE factor reduces \( F / 0.0006 \) direction (unidirectional output (i.e., collection efficiency for a 10 (20) unit-cell waveguide in a both sides, the collection efficiency in the traction. For the case of a waveguide surrounded by air on

enhancement is still very significant. (smaller waveguide) remains suitable as a single-photon moment, e.g., 100 D (solid curve), then only the 10 unit cell (smaller waveguide) remains suitable as a single-photon source, since the 20 unit-cell waveguide already yields strong-coupling (cf. the spectral doublet in the QD permittivity response). We next add in an integrated wire waveguide-coupler (non-PC), which is known to efficiently couple to a fiber [23], and example the case of 10 unit cells only [cf. Figs. 3(e) and 3(f)]. While the presence of the coupler (solid curve) reduces \( F \), the overall SE enhancement is still very significant.

Finally, we address the important issue of photon extraction. For the case of a waveguide surrounded by air on both sides, the collection efficiency in the \( x \) direction for a 10 (20) unit-cell waveguide is 26% (27.5%) in either direction (\( \pm x \)) for air on both sides, with the rest being lost to vertical leakage. However, with a PC mirror, the collection efficiency for a 10 (20) unit-cell waveguide in a unidirectional output (i.e., \( -x \) direction, to the left) is 53% (52%). More importantly, with the integrated coupler, more than 60% of coupling into the output waveguide mode is estimated (this also includes a mode-overlap calculation), which can easily be integrated with an output fiber coupler [23]. For larger waveguides such as with 20 unit cells, this output coupling was actually found to decrease, signaling that more light is lost vertically for longer waveguides, even for an ideal structure. Although we have not yet tried to optimize the material parameters, we anticipate that with suitable design, optimum SE factors for single photon emission can be achieved with a simultaneous 70%–80% output coupling efficiency.

In conclusion, we have studied the physics of single-QD photon emission from finite-size PC waveguides and proposed a miniaturized device that merits investigation for single-photon emission applications “on chip.” Very large SE factors are obtained, and efficient unidirectional output is achieved using a PC mirror and an integrated output coupler. The single-photon emitter is expected to be robust with respect to fabrication disorder and is small enough to allow deterministic positioning of a single QD.

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27. We use FDTD Solutions, see: www.lumerical.com.