1. energy released is:
   - mass of $^{40}$K nucleus
   - minus mass of $^{40}$Ar nucleus
   - minus mass of positron (same as electron mass $m_e$)
   - minus mass of neutrino (ignore)

   now, if we are using atomic masses, then:
   - mass $^{40}$K atom = mass of $^{40}$K nucleus + 19 $m_e$ – binding energy (ignore)
   - mass $^{40}$Ar atom = mass of $^{40}$Ar nucleus + 18 $m_e$ – binding energy (ignore)

   and we always find atomic mass values in tables, so using that, energy released $Q$ is:
   - mass $^{40}$K atom – 19 $m_e$
   - minus (mass $^{40}$Ar atom – 18 $m_e$)
   - minus $m_e$

   $Q = [m(^{40}\text{K}) - m(^{40}\text{Ar}) - 2 m_e] c^2$

   $m(^{40}\text{K}) = 39.963998475$ amu (from the NNDC table)
   $m(^{40}\text{Ar}) = 39.96238312251$ amu (from the NNDC table)
   difference = 0.00161535 amu or 1.5046888329 MeV,
   since 1 amu = 931.494 MeV/c$^2$

   $Q = 1.5047 - 2 (0.511) = 0.483$ MeV

   The energy released in the positron decay of $^{40}$K is 0.483 MeV.

2. Energy loss using the Bethe formula:

   $-\frac{dE}{dx} = \frac{4 \pi k^2 e^4 z^2}{m_e c^2} \frac{\rho Z N_A}{A} B(v) = 4 \pi r_e^2 m_e c^2 z^2 \frac{\rho Z N_A}{A} B(v)$

   where $r_e$ is 2.818e–13 cm, $m_e c^2 = 0.511$ MeV
   $z^2=1$ (for a muon)
   $\gamma = 2000/105.66 = 18.92864$
   $\beta = v/c = 0.9986$ using $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$

   since this is $dE/dx$ for a compound (water $\text{H}_2\text{O}$) we must calculate the effective $Z$, effective $A$ and the effective $I_{\text{eff}}$
   $Z_{\text{eff}} = 10$, $A_{\text{eff}} = 18$ g/mol, $N_A = 6.022e23$ atoms/mol

   $\ln I_{\text{eff}} = \sum a_i Z_i \ln I_i / Z_{\text{eff}}$

   $\ln I_{\text{eff}} = 2(1) \ln 19.2 / 10 + 1(8) \ln 95.0 / 10 = 4.234$ and
   $I_{\text{eff}} = 69.0$ eV
Using this formula for $B(v)$, which includes relativistic effects and $W_{\text{max}}$ being the maximum energy transfer accounting for finite $M$ and $m_e$:

$$W_{\text{max}} = \frac{2m_e v^2 \gamma^2}{1 + 2 \frac{m_e}{M} \sqrt{1 + \gamma^2 \beta^2} + \left(\frac{m_e}{M}\right)^2}; \quad \text{if } M \gg m_e, W_{\text{max}} = 2m_e v^2 \gamma^2$$

$$B(v) = \frac{1}{2} \ln\left(\frac{2m_e v^2 \gamma^2 W_{\text{max}}}{I^2}\right) - \beta^2$$

plugging everything into the formula: $W_{\text{max}} = 308.637$ MeV

[aside: if I just calculate using $W_{\text{max}} = 2m_e v^2 \gamma^2$ is would get 365.151 MeV, so the full calculation does make a difference (and it’s mostly the relativistic part)]

plugging everything into $B(v)$:

$B(v) = 0.5 \ln \left[ \frac{(365.151 \text{ MeV}) (308.637 \text{ MeV})}{(69.0 \text{ eV})^2} \right] - (0.9986)^2$

$B(v) = 15.3976 - 0.9972 = 14.4004$

So, finally $-dE/dx = 4(\pi)(2.818 \times 10^{-13} \text{ cm})^2 (0.511 \text{ MeV}) (1/0.9986^2) (1 \text{ g/cm}^3) 10/(18 \text{ g/mol}) 6.022 \times 10^23 (\text{mol}^{-1}) (14.4)$

$= 2.46 \text{ MeV/cm}$

3. Radiation dose from eating 10 kBq $^{210}\text{Po}$ source:

10,000 decays per second depositing the full 5.3 MeV per decay

$= 5.3 \times 10^9$ eV per second and this is distributed in the body of 70 kg, so that’s

$= 0.0757 \times 10^7$ eV/kg per second $= 1.2 \times 10^{-3}$ J/kg per second or Gy per second

use a biological weighting factor of 20

$= 2.426 \times 10^{-9}$ Sv per second

[aside: that’s 8.7 µSv per hour, a radiation dose much less than levels reported around the Fukushima nuclear plant which were in the many mSv per hour...the problem with eating this source though is that it stays in the body and delivers the dose over quite a long period of time]

$\lambda_{\text{decay}} = \ln 2/t_{1/2} = \ln 2/138 = 0.0050 \text{ d}^{-1}$

$\lambda_{\text{biological}} = \ln 2/t_{\text{bio}} = \ln 2/30 = 0.0231 \text{ d}^{-1}$

thus, the total $\lambda = 0.028 \text{ d}^{-1}$

Total integrated dose: $\int_0^{20} A_0 e^{-\lambda t} dt = \frac{A_0}{\lambda} \left[1 - e^{-\lambda(20)}\right] = \frac{A_0}{\lambda} [0.4302]$

Where $A_0 = 2.426 \times 10^{-9}$ Sv/s and total $\lambda = 0.028 \text{ d}^{-1} = 3.2555 \times 10^{-7} \text{ s}^{-1}$

Total integrated dose $= 2.426 \times 9 / 3.2555 \times 7 \times [0.4302] = 0.0032 \text{ Sv or 3.2 mSv}$
So, that’s a total dose equal to the average annual dose a person receives from natural and man-made radiation and radioactivity. This is probably not enough $^{210}\text{Po}$ to poison someone (like that Russian spy did).

$10\text{ kBq} = \lambda_{\text{decay}} \times N$, where $\lambda_{\text{decay}} = \ln \frac{2}{t_{1/2}} = 0.0050\text{ d}^{-1}$ or $5.8\times10^{-8}\text{ s}^{-1}$ and solve for $N = \frac{10000}{5.787\times10^8} = 1.72\times10^{11}$ atoms of $^{210}\text{Po}$

$1.72\times10^{11}/6.022\times10^{23} \times 210\text{ g/mol} = 6.0\times10^{-11}$ or $0.06\text{ ng}$, a very tiny amount indeed.