Improving Signal-to-Noise

Measurement Systems

block diagram of a measurement system

- physical effect
- transducer
- signal conditioning electronics
- e.g. amplifiers, filters
- signal transmission line
- analog-to-digital conversion
- ADC could be here
- digital signal processing
- data acquisition and display
- signal conditioning electronics
- e.g. amplifiers, filters

noise may enter or may be present in any element of the system

we will examine almost all of these aspects in this course
Improving S/N: General Concept

- we’ve observed that noise power increases with bandwidth $\Delta f$ for both Johnson and shot noise
- we’ve heard about $1/f$ noise (power spectrum) and that it is common
- therefore, basic techniques for noise reduction
  - limit bandwidth $\rightarrow$ filter
  - shift signal to higher frequency where $1/f$ noise power is smaller
  - lock-in on a specific frequency in the signal
    - perhaps “artificially” introduced
    - ...and this can accomplish both limiting bandwidth and shifting to higher $f$

Amplifiers

- we won’t discuss amplifiers much
  - you’ve seen in your electronics class(es)
- typically transducer signals aren’t large enough to be read out directly; an amplifier is needed
- in general, does an ordinary, standalone amplifier improve S/N?
  - amplifies both signal and noise, so no it doesn’t
  - in fact, makes it worse because it adds its own noise
    - Johnson noise, shot noise, $1/f$ noise thought to be from charge trapping/releasing in transistors in the amplifier

*Figure 1: Frequency Characteristic of Op Amp Noise*
Multi-Stage Measuring Systems

In general, a measurement system consists of a measuring device followed by an amplifier chain. Each of these units will introduce noise, but it is of interest to consider which part of the chain must be designed with maximum care.

![Diagram of multi-stage measuring system]

- Signal amplitude, $S_2 = G_1 G_2 S_0$
- First stage noise, $N_1 = G_1 \sqrt{N_0^2 + n_1^2}$
- Second stage noise, $N_2 = G_2 \sqrt{N_1^2 + n_2^2}$

Hence

$$N_2 = G_2 \sqrt{G_1^2 (N_1^2 + n_1^2) + n_2^2}$$

Rearranging

$$N_2 = G_2 G_1 n_1 \sqrt{1 + \frac{n_1^2}{N_0^2} + \frac{n_2^2}{G_1^2 N_0^2}}$$

Low Noise, High Gain First Stage

- After two amplifier stages:
  $$\frac{S_2}{N_2} = \frac{S_0}{N_0} \left(1 + \frac{n_1^2}{N_0^2} + \frac{n_2^2}{G_1^2 N_0^2}\right)^{1/2}$$
  $$\therefore S_2 / N_2 < S_0 / N_0$$

- It's clear that an amplifier doesn't improve signal-to-noise → it is worse because of the noise added by the amplifier itself

- If the first stage gain is large, noise in later stages are less important

- Conclusion: when designing a measurement system, put lots of effort into the first stage (preamplifier), designing it for low noise and high gain
Preamp Considerations

- reduce noise by cooling:
  - e.g. $R = 1\, \Omega$; $T = 300\, K, 77\, K, 4.2\, K$; $\Delta f = 1\, Hz$

<table>
<thead>
<tr>
<th>$T$</th>
<th>$V_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>300K</td>
<td>4 nV</td>
</tr>
<tr>
<td>77K</td>
<td>2 nV</td>
</tr>
<tr>
<td>4.2K</td>
<td>0.5 nV</td>
</tr>
</tbody>
</table>

- type of component:
  - junction transistors have shot noise (minority charge carriers crossing the junction is a statistical process)
  - field effect transistors behave as variable resistance in the channel (no shot noise, apart from small gate leakage current)

- $1/f$ noise in resistors:

<table>
<thead>
<tr>
<th>Resistor Type</th>
<th>Noise Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>carbon composition</td>
<td>0.1-3.0 nV</td>
</tr>
<tr>
<td>carbon-film</td>
<td>0.05-0.3 nV</td>
</tr>
<tr>
<td>metal-film</td>
<td>0.02-0.2 nV</td>
</tr>
<tr>
<td>wire-wound</td>
<td>0.01-0.2 nV</td>
</tr>
</tbody>
</table>

$V_n = \sqrt{4kT\Delta f}$

1 V applied, integrated over one decade of frequency

![First stage preamplifier.](image)

**Fig. 4.14** First stage preamplifier.

electrical model of an amplifier

**Measurement of DC Signals**

- Johnson and shot noise increase with bandwidth $\Delta f$
- to improve signal-to-noise of a DC measurement, you want a low-pass filter
- e.g. passive RC low-pass filter

\[
\frac{V_{out}}{V_{in}} = \frac{-j}{\omega C} \left( \frac{R + j}{\omega C} \right) = \frac{1 - j/\omega RC}{\omega^2 R^2 C^2 + 1}
\]

\[
A = \frac{V_{out}}{V_{in}} = \frac{\sqrt{1+(\omega RC)^2}}{1+(\omega RC)^2} = \frac{1}{\sqrt{1+(\omega RC)^2}}
\]

\[
\tan \theta = (\omega RC) \quad \text{phase delay}
\]

\[
\omega_0 = \frac{1}{RC} \quad \text{3 dB frequency}
\]

**remember dB always refers to power and not voltage**
RC Filter as an Integrator

- AC signal applied to RC filter, in steady-state: \( I = \frac{V_m}{R + j\omega C} \)
- For large signal frequency: \( \omega \gg \omega_0 \) \( \Rightarrow \) \( R \gg \frac{1}{\omega C} \)
  \[ I = \frac{V_m}{R} \quad v_c = \frac{q}{C} = \frac{1}{C} \int i \, dt = \frac{1}{C} \int \frac{V_m}{R} \, dt = \frac{1}{RC} \int v_m \, dt \]
- So for \( \omega \gg \omega_0 \) or \( \frac{T}{\tau} = \frac{2\pi/\omega}{\tau} = \frac{1}{RC} \) period much less than RC time constant
  - The output signal is the integral of the input signal
  - RC integrator functions as an averager over \( \tau \gg T \)
  - What’s happening is averaging and smoothing the short period noise
- Large \( \tau \) is small \( \omega_0 \) for a low-pass filter (i.e. small bandwidth)

Measurement of DC cont’d

- Low-pass filtering is integration or averaging
  - The capacitor accumulates charge by “integrating” the current flow
  - The longer the integration time, the smaller the bandwidth
  \[ \Delta f = \frac{1}{2T} \]
- Potential difficulty with DC measurements (low frequency)
  - 1/f noise can be present, resulting in lower S/N
Measurement of Signals at Some Finite Frequency

- the signal you are wanting to measure has frequency content that is above the region of 1/f noise?
- if yes, apply a bandpass filter around the desired frequency
- if not, move the signal content to a higher frequency range (where the noise power is lower) by modulating it

Chopping Signals

- optical or electronic choppers modulate the signal
- Fourier transform of the product of two functions is the convolution of their individual transforms
  - shifts the signal content up to the carrier (modulation) frequency
- more on Fourier transforms and signals in the next lecture
  - \( V_{\text{out}} = V_{\text{in}} \times K(t) \)
  - \( \tilde{V}_{\text{out}}(\omega) = \tilde{V}_{\text{in}}(\omega) \otimes \tilde{K}(\omega) \)
AC Driven Measurement Technique

DC signal may be very weak; 1/f noise from the amplifier may be too much

AC driven measurement modulates the signal above the 1/f corner frequency of the amplifier

Lock-in Amplifiers

- a particular technique of picking up a modulated weak signal (and imposing a very tight bandpass filter) is the lock-in amplifier
First, You Need a Phase Sensitive Amplifier

- if the signal has a known frequency, you can pick off that frequency with a phase-sensitive amplifier
- makes for a very narrow bandpass to strongly reject noise at all other frequencies

Phase Sensitive Output

- same frequency AND phase required for non-zero output

---

turn a knob to adjust the phase delay (and feed in the reference frequency or turn knob until matches the signal frequency)

---

Fig. 4.30 The output of a phase sensitive detector: (i) when signal and reference have similar frequency and phase, (ii) when the phase of the signal frequency is shifted by 90°, and (iii) when there is no correlation between the signal and reference.
Mathematically Speaking

- this is cross correlation by an analog circuit
- input signal is f(t)
- reference signal is g(t) with phase shift $\delta$
- integrate over period $T = RC$
- output is the cross correlation coefficient, a function of $\delta$ and $T$

$$R(\delta, T) = \frac{1}{T} \int_{0}^{T} f(t) \cdot g(t + \delta) \, dt$$

Lock-In Amplifier

- introduce periodicity in signal artificially
  - e.g. with a chopper
  - send a reference chopped signal into the lock-in
- or send the reference oscillator to electronically modulate the signal
Chopping the Signal

- introduces periodicity in the signal
- reference signal can be synthesized from the chopper also, ensures frequency match
- also possible to electronically chop a signal
- but, don’t electronically chop the noise (i.e. modulate before any amplifiers)

Phase Shifter

- analog circuit to shift phase

you don’t need to know the details just know that an analog circuit to shift phase is easy to build
Small Frequency Shift

- signal and reference differ by $\Delta \omega$

$$\cos(\omega_1 t) \cos(\omega_2 t) = \frac{\cos((\omega_1 + \omega_2) t) + \cos((\omega_1 - \omega_2) t)}{2}$$

- only if $\Delta \omega$ is small compared to $1/RC$ will the signal pass the low-pass filter
  - the integrator or averager gives a non-zero signal if integrating over less than one period $= 2\pi/\Delta \omega$

Lock-In Amplifier Bandwidth

- set by the RC time constant of the integrator
- $\Delta f = 2 f_c = 2 \times \frac{1}{2\pi RC} = \frac{1}{\pi RC}$
  - i.e. the passband is both sides of the lock-in operating frequency

![Diagram](image.png)  
Fig. 4.33 The effective bandwidth of a lock-in amplifier is $\pm 1/(2\pi RC)$. 
Example: Lock-In Bandpass

- measuring frequency 10 kHz
- RC = 1 s
- $\Delta f = \frac{1}{\pi} = 0.3$ Hz
  - detecting signals between 9999.7 and 10000.3 Hz
- $Q = \frac{f}{\Delta f} = 30,000$ or so
- huge noise reduction and high Q-factor

A lock-in amplifier is a way to implement a high-Q bandpass filter by utilizing a phase-sensitive amplifier to cross-correlate the modulated signal with a reference signal, generating a difference which can be low-pass filtered easily (and with high quality)

- in comparison, narrow bandpass analog circuits are more complicated to build with similar quality
- in comparison, narrow bandpass digital filters require the use of ADC and digital signal processing

Pros and Cons

- very narrow bandwidth rejects noise, allows measuring very weak signals in noisy ambient conditions
  - e.g. can modulate and lock-in on a weak optical signal and not be affected by room ambient lighting
- measurement frequency shifted from DC to a higher value reducing effect of 1/f noise in the measurement system electronics
- disadvantage is the slow response time, also set by RC, such that tuning the bandwidth for smaller bandwidth thus larger RC slows the response of the system
  - e.g. RC = 1 s, takes maybe up to 5 RC for signal to reach steady-state after change in signal level, or 5 s