Coupled-Cavity QED Using Planar Photonic Crystals

S. Hughes*

Department of Physics, Queen's University, Kingston, Ontario K7L 3N6 Canada

(Received 16 August 2006; published 23 February 2007)

We introduce a technique for controlling cavity QED by indirectly coupling two planar-photonic-crystal nanocavities through an integrated waveguide. Guided by an explicit analytical expression for the photon Green function, the resulting optical response of a single quantum dot, embedded in one of the cavities, is shown to be profoundly influenced by the distant cavity. The regimes of cavity QED, e.g., vacuum Rabi splitting, are made significantly easier and richer than with one cavity alone.

DOI: 10.1103/PhysRevLett.98.083603 PACS numbers: 42.50.Pq, 41.20.Jb, 42.25.Bs, 42.55.Tv

The ability to tailor the optical response of an atom or quantum dot (QD) has been intriguing scientific researchers for some time. Since the early work of Purcell [1], it has been known that the surrounding environment of an atom in which a photon is emitted can change the spontaneous emission rate, and thus the emission characteristics are not solely dependent on the property of the emitter. Further motivation has been sparked by the introduction of photonic crystals (PC's) [2,3] since they have the means to control the surrounding vacuum of electromagnetic modes, and thus the Purcell effect can occur over broadband frequencies [4]. For strong coupling of an atom to a cavity, the enhanced emission rate associated with a single resonance can result in a doublet in the frequency regime, whose spectral width corresponds to the vacuum Rabi splitting in the language of quantum optics. In the domain of semiconductor optics, several recent breakthroughs have occurred including single QD vacuum Rabi splitting [5–7], very large Q/V ratio nanocavities [8], and the deterministic coupling of QD's to planar PC's [9].

For applications in cavity QED, the current trend seems to be to increase the single-cavity Q to as large a value as possible. On the other hand, there is a rich degree of flexibility that exists by combining waveguides and cavities; e.g., quantum networks through fibers and nanodots have been proposed using cavity-assisted Raman processes [10]. In this Letter we investigate light-matter interactions using a single QD within a fully integrated planar PC platform that contains two cavities and one waveguide. Importantly, the material is extended from the usual cavity to include a distant (no direct coupling) cavity with similar characteristics, *indirectly* coupled through a waveguide. We term this new light-matter regime: *coupled-cavity QED*.

The structure of interest is depicted in Fig. 1, showing a planar PC coupled to two nanocavities. The QD is embedded in cavity 1. The geometry is somewhat complicated and little insight will be gained by running large scale numerical calculations. Instead we motivate the fact that the essential physics can be made clear by seeking out the photon Green function tensor (GFT). Armed with the analytic GFT of the entire structure, we show that under certain conditions, arbitrarily large enhancements of the local photon density of states can be obtained, regardless of the individual cavity Q. More generally, we investigate a novel nanophotonics system that connects to both classical and quantum electromagnetically induced transparency (EIT)-like phenomena using single QD's, opening up a rich range of light-matter control as a very fundamental level. The electric-field GFT $\mathbf{G}(\mathbf{r}, \mathbf{r}')$ (the ω dependence we assume is implicit) describes the field response at $\mathbf{r}'$ to an oscillating dipole at $\mathbf{r}$, and can be defined from Maxwell equations. The GFT describes many fundamental photon coupling effects, including dipole-dipole coupling [11], the Purcell effect [4], QED near an interface [12], optical scattering loss [13], and the full machinery of non-Markovian quantum optics [14], and, so upon solution, the GFT can highlight the underlying physics in an elegant and transparent way.

We first derive the propagating electric field and the GFT for the PC waveguide and one cavity. This solution set is termed $\mathbf{E}_{wc}(\mathbf{r})$ and $\mathbf{G}_{wc}(\mathbf{r}, \mathbf{r}')$, with the subscript wc referring to the waveguide and cavity under investigation. We consider waveguide and cavity resonances that are deep inside the PC band gap and a propagating waveguide mode that is below the light line (lossless). The GFT is defined by

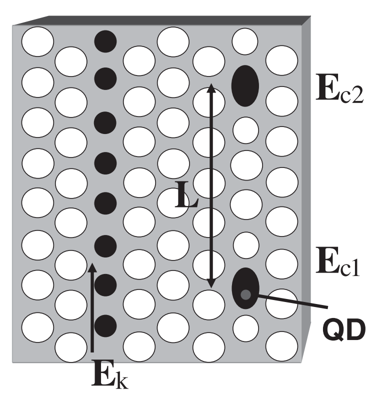

FIG. 1. Simple schematic showing indirect coupling between two photonic crystal cavities connected through an integrated waveguide. Cavity 1 contains an embedded QD. The propagating Bloch mode is labeled as $\mathbf{E}_k$, while the cavity modes are $\mathbf{E}_{c1}$ and $\mathbf{E}_{c2}$, that are spatially separated by L.

 © 2007 The American Physical Society

expanding into all the electric-field modes $\mathbf{G}(\mathbf{r},\mathbf{r}') = \sum_{\alpha,\beta} A_{\alpha\beta}\mathbf{E}_\alpha(\mathbf{r})\mathbf{E}_\beta(\mathbf{r}')$, where $A_{\alpha\beta}$ is the expansion coefficients that we seek to obtain, and $\mathbf{E}_{\alpha,\beta}$ represent the modes of the PC system that may belong to the cavity or waveguide, with these modes weakly coupled to each other. We adopt shorthand notation: $\mathbf{E}(\mathbf{r}) \to |\mathbf{E}\rangle$, $\varepsilon_t(\mathbf{r}) \to \hat{\varepsilon}_t$, $\mathbf{G}(\mathbf{r},\mathbf{r}') \to \hat{\mathbf{G}}$, $\delta(\mathbf{r}-\mathbf{r}') \to \hat{\mathbf{1}}$, $\hat{V} \to \Delta\varepsilon(\mathbf{r})$, with the latter a material permittivity change that drives a GFT solution. With a separation of the quasitransverse and quasilongitudinal electromagnetic modes, the GFT $\hat{\mathbf{G}} = \sum_{\mathbf{k}}[\omega^2/(\omega_{\mathbf{k}}^2-\omega^2)|\mathbf{E}_{\mathbf{k}}^T\rangle\langle\mathbf{E}_{\mathbf{k}}^T| + |\mathbf{E}_{\mathbf{k}}^L\rangle\langle\mathbf{E}_{\mathbf{k}}^L|]$. It proves advantageous to define a new GFT, $\hat{\mathbf{K}} = \sum_{\mathbf{k}}\omega_{\mathbf{k}}^2/(\omega_{\mathbf{k}}^2-\omega^2)\times|\mathbf{E}_{\mathbf{k}}\rangle\langle\mathbf{E}_{\mathbf{k}}|$, where the modes from now on are all generalized transverse ($|\mathbf{E}_{\mathbf{k}}\rangle = |\mathbf{E}_{\mathbf{k}}^T\rangle$) and satisfy $\nabla\times\nabla\times|\mathbf{E}_{\mathbf{k}}\rangle = \omega_{\mathbf{k}}^2/c^2\hat{\varepsilon}_t|\mathbf{E}_{\mathbf{k}}\rangle$. The full GFT then becomes $\hat{\mathbf{G}} = \hat{\mathbf{K}} - \hat{1}/\hat{\varepsilon}_t$, which is identical to the GFT's discusssed by Wubs *et al.* [15], within a multiple scattering formalism.

With a background permittivity $\hat{\varepsilon}_{pc}$ and a waveguide defect perturbation $\hat{V}_w$, the waveguide modes are normalized from $\langle\mathbf{E}_\alpha|\hat{\varepsilon}_w|\mathbf{E}_\beta\rangle = \delta_{\alpha\beta}$, and $\sum_\alpha\langle\mathbf{E}_\alpha|\hat{\varepsilon}_w|\mathbf{E}_\alpha\rangle = \hat{1}$, where the total permittivity is $\hat{\varepsilon}_w = \hat{\varepsilon}_{pc} + \hat{V}_w$. The propagation modes with wave vector k can be scaled as $|\mathbf{E}_k\rangle = \sqrt{a/L_w}|\mathbf{e}_k\rangle e^{ikx}$, with L_w the length of the waveguide, a the pitch and $|\mathbf{e}_k\rangle$ the periodic Bloch mode. One finds the waveguide $\hat{\mathbf{G}}_w = \sum_k\omega_k^2/(\omega_k^2-\omega^2)|\mathbf{E}_k\rangle\langle\mathbf{E}_k| - \hat{1}/\hat{\varepsilon}_w$. Similar arguments apply for the PC nanocavity with modes normalized in the usual way, and the cavity $\hat{\mathbf{G}}_c = \omega_c^2/(\omega_c^2 - \omega^2)|\mathbf{E}_c\rangle\langle\mathbf{E}_c| - \hat{1}/\hat{\varepsilon}_c$, where ω_c is the cavity resonance and $\hat{\varepsilon}_c = \hat{\varepsilon}_{pc} + \hat{V}_c$ ($\hat{V}_c$ is the cavity perturbation). We exploit the Dyson equation $\hat{\mathbf{G}} = \hat{\mathbf{G}}_0 + \hat{\mathbf{G}}_0\hat{V}\hat{\mathbf{G}}$, and expand out the total GFT defined earlier. Subsequently, the main task is to invert a matrix M, with elements $M_{c,c} = (\omega_c^2-\omega^2)/\omega_c^2$, $M_{k,k} = (\omega_k^2-\omega^2)/\omega_k^2$, and $M_{k,c} = \langle\mathbf{E}_k|\hat{V}_c|\mathbf{E}_c\rangle = V_c^{k,c} \equiv V^{k,c}$, where we have used the orthogonality relation $\langle\mathbf{E}_\alpha|\hat{\varepsilon}_t|\mathbf{E}_\beta\rangle = \delta_{\alpha,\beta}$ [16]. It follows that

$$\hat{\mathbf{G}}_{wc} = \hat{\mathbf{K}}_{wc} - \frac{\hat{1}}{\hat{\varepsilon}_t} = \frac{\hat{\mathbf{K}}_w\hat{V}_c|\mathbf{E}_c\rangle\langle\mathbf{E}_c| + \omega_c^2|\mathbf{E}_c\rangle\langle\mathbf{E}_c|}{\omega_c^2 - \omega^2 - \langle\mathbf{E}_c|\hat{V}_c\hat{\mathbf{K}}_w\hat{V}_c|\mathbf{E}_c\rangle} - \frac{\hat{1}}{\hat{\varepsilon}_t}, \quad (1)$$

and similarly for the electric field: $|\mathbf{E}_{wc}\rangle = |\mathbf{E}_{k_h}\rangle + \hat{\mathbf{G}}_{wc}\hat{V}_c|\mathbf{E}_{k_h}\rangle$, where $|\mathbf{E}_{k_h}\rangle$ is the injected field. The cavity field ($|\mathbf{E}_c\rangle$) belongs to either a localized defect mode in cavity 1 ($|\mathbf{E}_1\rangle$) or cavity 2 ($|\mathbf{E}_2\rangle$). Similar expressions have been obtained by Cowan and Young [16] and by Hughes and Kamada [17], using a similar technique but without any direct consideration for the longitudinal modes. The subtle differences is that there appears ω_c^2 rather than ω^2 before the cavity contribution and a term that will include self-energy corrections at the cavity perturbation.

Next we add in the second cavity (cavity 2), separated by a distance $+L$ along the waveguide, taken to be an integer number of unit cells along the waveguide. We consider indirect coupling between cavities. Employing the appropriate Dyson equation, $\hat{\mathbf{G}}_{w2} = \hat{\mathbf{G}}_0 + \hat{\mathbf{G}}_0\hat{V}_2\hat{\mathbf{G}}_{w2}$, and $|\hat{\mathbf{E}}_{k_h}\rangle \to (1 + \hat{\mathbf{G}}_{w2}\hat{V}_2)|\hat{\mathbf{E}}_{k_h}\rangle$, as the solutions prior to adding in the first cavity (so $\hat{\mathbf{G}}_0 = \hat{\mathbf{G}}_w$ is for the waveguide alone), we obtain

$$\hat{\mathbf{G}}_{wcc} = \frac{(1+\hat{\mathbf{K}}_{w2}\hat{V}_2)\hat{\mathbf{K}}_w\hat{V}_1|\mathbf{E}_1\rangle\langle\mathbf{E}_1| + \omega_1^2|\mathbf{E}_1\rangle\langle\mathbf{E}_1|}{\omega_1^2 - \omega^2 - \langle\mathbf{E}_1|\hat{V}_1(1+\hat{\mathbf{K}}_{w2}\hat{V}_2)\hat{\mathbf{K}}_w\hat{V}_1|\mathbf{E}_1\rangle} - \frac{\hat{1}}{\hat{\varepsilon}_t} \quad (2)$$

and

$$|\mathbf{E}_{wcc}\rangle = (1+\hat{\mathbf{K}}_{w2}\hat{V}_2)|\mathbf{E}_{k_h}\rangle + \frac{(1+\hat{\mathbf{K}}_{w2}\hat{V}_2)\hat{\mathbf{K}}_w\hat{V}_1|\mathbf{E}_1\rangle\langle\mathbf{E}_1|\hat{V}_1(1+\hat{\mathbf{K}}_{w2}\hat{V}_2)|\mathbf{E}_1\rangle + \omega_1^2|\mathbf{E}_1\rangle\langle\mathbf{E}_1|\hat{V}_1|\mathbf{E}_{K_h}\rangle}{\omega_1^2 - \omega^2 - \langle\mathbf{E}_1|\hat{V}_1(1+\hat{\mathbf{K}}_{w2}\hat{V}_2)\hat{\mathbf{K}}_w\hat{V}_1|\mathbf{E}_1\rangle} - \frac{(\hat{V}_1+\hat{V}_2)|\mathbf{E}_{k_h}\rangle}{\hat{\varepsilon}_t}, \quad (3)$$

where $\hat{\mathbf{K}}_{w2}$ is obtained from Eq. (1) using parameters that describe cavity 2 coupled to the waveguide on its own ($c = 2$). The total permittivity is now $\hat{\varepsilon}_t = \hat{\varepsilon}_{pc} + \hat{V}_w + \hat{V}_1 + \hat{V}_2$.

All that remains to be done is to carry out the complex integrations (whenever $\hat{\mathbf{G}}_w$ appears) that are aided by noting that the waveguide Bloch modes and the overlap integrals varying slowly with respect to k. We also assume mirror symmetry for the cavity mode along the propagation (x) direction. The total GFT can be written as $\hat{\mathbf{G}}_{wcc} = \sum_{\alpha,\beta}\hat{\mathbf{K}}_{wcc}^{\alpha,\beta} - \hat{1}/\hat{\varepsilon}_t$, with, for example,

$$\hat{\mathbf{K}}_{wcc}^{1,1} = \frac{\omega_1^2|\mathbf{E}_1\rangle\langle\mathbf{E}_1|}{\omega_1^2 - \omega^2 - i\omega\Gamma_1^0 - i\omega\Gamma_1^c[1 + e^{i2k_\omega L}r_2(\omega)]}, \quad (4)$$

where $r_2(\omega) = i\omega\Gamma_2^c/[\omega_2^2 - \omega^2 - i\omega(\Gamma_2^0 + \Gamma_2^c)]$ is the reflection coefficient of cavity 2. Since the PC cavity modes are strongly confined, the above is the dominant contribution for spatial points near cavity 1. The cavity-to-waveguide coupling rate is $\Gamma^c = \omega^2 L_w/2v_g|V^{k_\omega,c}|^2$, where v_g is the photon group velocity and the overlap integral $V^{k_\omega,c} = \langle\mathbf{E}_{k_\omega}|\hat{V}_c|\mathbf{E}_c\rangle$. The coupling to radiation modes above the light line is included with the bare cavity coupling rate, Γ^0, which is typically much smaller that the cavity rate.

The aforementioned self-coupling GFT [Eq. (4)], describing cavity 1, already contains some remarkable new physics. Assuming a QD within this environment with a dipole moment $\mathbf{d} = d\hat{\mathbf{n}}_i$, with $\hat{\mathbf{n}}_i$ a unit vector in the direction of the dipole, it is well known that the radiative decay rate $\Gamma_r = \mathrm{Im}(\hat{\mathbf{K}}_{wcc})_{ii}2|d|^2/(\hbar\varepsilon_0)$, and the resonance shift $\Delta\omega = \mathrm{Re}(\hat{\mathbf{K}}_{wcc})_{ii}|d|^2/(\hbar\varepsilon_0)$. Note that we use $\hat{\mathbf{K}}$ rather than $\hat{\mathbf{G}}$ in the latter term to distinguish the frequency shift to a much smaller vacuum Lamb shift, that is of no interest here; we have also assumed weak coupling (for the moment) to connect to these formulas. Now consider the case of $\omega = \omega_1$ and $\Gamma_0 \approx 0$, with one cavity only, so that we recover the familiar expression $\hat{\mathbf{K}}_{wc} = iQ|\mathbf{E}_1\rangle\langle\mathbf{E}_1|$, with $Q = \omega_1/\Gamma_1^c$. Employing a cavity field value at the maximum field position, then $\hat{\mathbf{K}}_{wc} = iQ/[\varepsilon_t(r_d)V_{\text{eff}}]$. However, with two cavities as above we obtain $\hat{\mathbf{K}}_{wcc} = iQ/[(\varepsilon_t(r_d)V_{\text{eff}})(1 - e^{i2k_\omega L})]$ so that both the decay rate and the resonance shift can obtain substantially different values than with 1 cavity alone. As an example, with a phase factor $2k_\omega L = \pi/2 + 2n\pi$ $(n = 0, 1, 2\ldots)$, $\hat{\mathbf{K}}_{wcc} = \hat{\mathbf{K}}_{wc}/(1 - i)$; if $2k_\omega L = \pi + 2n\pi$ then $\hat{\mathbf{K}}_{wcc} = \hat{\mathbf{K}}_{wc}/2$; if $2k_\omega L = 3\pi/2 + 2n\pi$, $\hat{\mathbf{K}}_{wcc} = \hat{\mathbf{K}}_{wc}/(1 + i)$; and if $2k_\omega L = 2\pi + 2n\pi$ then $\hat{\mathbf{K}}_{wcc} = \infty$. So the second cavity (far away from the QD) can induce large frequency shifts, both positive and negative, reduce the local density of states by 2, or increase it indefinitely. Let us now see if these solutions make physical sense in terms of the required Bloch modes. For $2k_\omega L = \pi/2 + 2n\pi$ and taking $L = 10a$, then $k_\omega a = \pi/40 + n\pi/10$; taking, e.g., $n = 7$, then $k_\omega = 0.3625(2\pi/a)$ which is indeed a typical low loss propagation wave vector for planar PC waveguides. Similarly, if $2k_\omega L = 2\pi + 2n\pi$, $k_\omega a = (n + 1)\pi/10$, which is again easily satisfied [$n = 7$ gives $k_\omega = 0.4(2\pi/a)$]. We note in the latter case there is no decay, only an infinite resonance shift. In reality of course one must include a Γ^0 (as we do below), but the divergent nature and profound enhancements that we introduce here should still be observable, especially as it is well known that $\Gamma^0 \ll \Gamma^c$ for planar PC's. The k_ω values above typically yield group velocities of around $c/10$ depending very much on the design of the waveguide; importantly these should not be too close to the very slow light regime (near the band edge) which would result in a reduction of the cavity quality factor and produce large (disorder-induced) scattering losses [13,18].

We now introduce the single QD with a volume V_d and diameter much smaller than the wavelength of light. Thus the QD permittivity $\Delta\varepsilon^0 = \hat{\mathbf{n}}_i|d|^2/[2V_d\varepsilon_0\hbar(\omega_d - \omega - i\Gamma_d/2)]$ with Γ_d is the nonradiative decay rate, and ω_d the QD resonance frequency. In the narrow frequency range of interest, we need only consider the fundamental $1s$ exciton (electron-hole pair) resonance. With the QD at position r_d, once again we exploit a Dyson equation to obtain a renormalized QD permittivity $\Delta\varepsilon(r_d) = \hat{\mathbf{n}}_i|d|^2/[2V_d\varepsilon_0\hbar(\bar{\omega}_d - \omega - i\Gamma_d/2 - \Sigma^d)]$, where $\Sigma^d = V_d\langle r_d|\hat{\mathbf{K}}_{wcc}|r_d\rangle\Delta\varepsilon^0(\bar{\omega}_d - \omega - i\Gamma_d/2)$ depends on the background medium GFT (pre-QD) that we have just derived. Note that the main role of the depolarization, whose value depends on the QD and background permittivity [i.e., $(\hat{V}_1 + \hat{V}_2)/\hat{\varepsilon}_t$], is to give rise to a possible QD resonance shift. In principle, one should then worry about the shape and volume of the dot to make an accurate calculation. Instead, we assume the QD resonance will be renormalized already, so that $\bar{\omega}_d$ contains this electrostatic (frequency independent) contribution.

While the renormalized susceptibility is related to what one will observe above the cavity (via vertical leakage), the transmission of light yields a different and complimentary experimental signature of the regime under investigation. By injecting a forward propagating field (homogeneous solution with $k = k_h$) from the bottom of the first cavity, one obtains

$$t(\omega) = \frac{\langle x \to \infty|\mathbf{E}_{wcc}\rangle}{\langle x \to \infty|\mathbf{E}_{k_h}\rangle} = 1 + r_2(\omega) + \frac{i\omega\Gamma_1[1 + r_2(\omega)][1 + r_2(\omega)e^{i2k_\omega L}]}{\omega_1^2 - \omega^2 - i\omega\Gamma_1^0 - i\omega\Gamma_1^c[1 + r_2(\omega)e^{i2k_\omega L}] - \omega_1\Sigma^1}, \tag{5}$$

where $\Sigma^1 = \Omega^2/(\bar{\omega}_d - \omega - i\Gamma_d/2)$, with $\Omega^2 = \omega_1|d|^2\langle r_d|\mathbf{E}_1\rangle\langle\mathbf{E}_1|r_d\rangle/(2\varepsilon_0\hbar)$. At a peak antinode position, $\Omega^2 = \omega_1|d|^2/[2\varepsilon_0\varepsilon_t(r_d)\hbar V_{\text{eff}}]$. In the strong coupling regime, the transmitted (or reflected) spectrum from the first cavity alone (with the QD) will contain the familiar Rabi splitting of 2Ω. We remark that it is just as easy to calculate the reflection [17], but we focus here on the transmission as it is much easier to measure experimentally.

For calculations, we adopt the following PC and QD parameters: $\omega_d/(2\pi) \approx 230$ THz, $\Gamma_d = 1$ μeV [19], $\bar{\omega}_d = \omega_1$ (on resonance with cavity 1) and dipole moment $d = 30$ D($\sim 0.6e$ nm). The cavity Q factors are $Q^c = \omega_c/\Gamma^c = 4000$ and $Q^0 = \omega_c/\Gamma^0 = 80\,000$, that we assume to be the same in both cavities (the relaxation of this makes little qualitative difference on our findings and conclusions). For the cavity separation we take $L = 10a$ and choose 3 representative values of $2k_\omega L$, namely, $\pi/2 + 14\pi$, $\pi + 14\pi$, and $2\pi + 14\pi$. In Fig. 2 we present several example calculations of the PC-influenced permittivity and the light transmission. In (a) we consider 1 cavity only, yielding marginal splitting of the permittivity but a clear splitting of the transmitted field. Such an effect has recently been termed dipole-induced transparency [20], and interestingly can occur even when the QD is not in the strong coupling regime (cf. the thick solid curve); it should be noted that such effects were first anticipated in a reflection geometry [17]. In (b) the situation is drastically different, with substantial enhancements and reshaping of the QD resonance due to the external second cavity. In particular, with changing phases, one goes from a large asymmetric permittivity (note that these peaks are reversed with $2k_{\omega_1}L = 3\pi/2 + 14\pi$), to a single peak only but enhanced (c), through to a pronounced splitting with a significantly enhanced Rabi splitting (d). In this latter regime, there are now two spectral positions that exhibit EIT-like phenomena. It is important to stress the difference between classical EIT, single QD-cavity EIT, and the coupled-cavity EIT. The former can never connect to anything quantum and is limited by the finite broadening of the bare cavity; the second depends on the QD broadening as well as the properties of the bare cavity; however, the latter is a mixed case, but one in which the local photon density

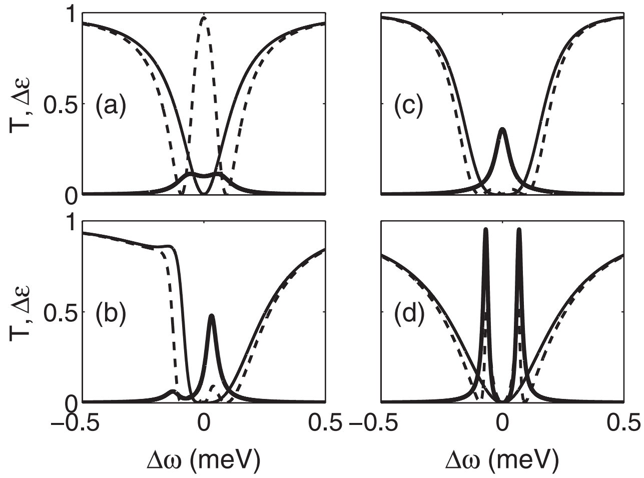

FIG. 2. (a) Absolute value of the quantum dot (QD) permittivity ($\Delta\epsilon$) and light transmission ($T = |t|^2$) with one cavity only; the QD is resonant with the cavity mode. The thin solid and chain curves display the transmission without and with a QD, respectively, while the thick solid curve shows the permittivity. (b)–(d) As in (a) but with a spatially separated cavity with various phase separation parameters (see text): $2k_\omega L = \pi/2 + 14\pi$, $2k_\omega L = \pi + 14\pi$, and $2k_\omega L = 2\pi + 14\pi$, respectively.

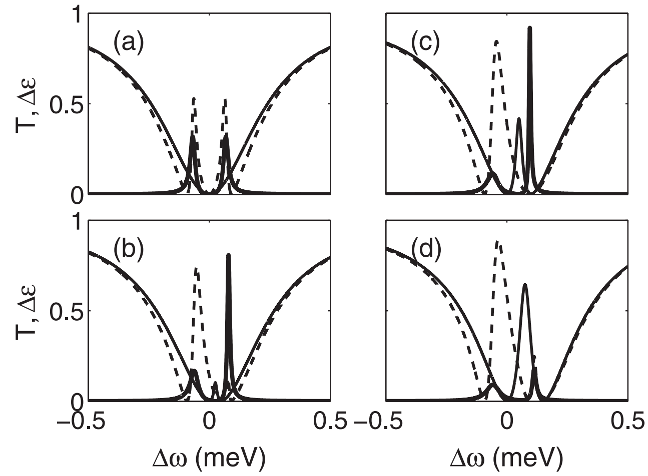

FIG. 3. (a) As in Fig. 2(d) with two spatially separated cavities, but the permittivity has been rescaled down by a factor of 3 (see text). (b)–(d) As in (a) but with a detuning $\omega_2 - \omega_1$ of 0.05, 0.1, and 0.1 meV, respectively.

of states can be made arbitrarily large and in which frequency shifts can be introduced into the one photon emitter; with different cavity frequencies, several peaks can also be obtained.

Finally in Fig. 3 we study the effect of detuning between the two cavities ($\omega_1 \neq \omega_2$). The first panel (a) displays the nominal case as a reference [identical to Fig. 2(d) with no detuning], while (b)–(d) show similar calculations but with $\omega_2 - \omega_1 = 0.05$, 0.1, and 0.15 meV, respectively. Note that we have reduced (rescaled) the permittivity by a factor of 3 here in comparison to the previous figure to accommodate the massive increases that occur due to detuning. A large resonant enhancement occurs as one of the Rabi sidebands overlaps with the second cavity's resonance (near 0.1 meV) resulting in an order-of-magnitude enhancement of the QD permittivity over 1 cavity alone. In addition, there are substantial frequency shifts of the QD resonance of the order 0.04 meV [see (c) and (d)], yielding Fano line shapes due to coupling between a narrow resonance and a continuumlike system.

In summary, a coupled-cavity QED regime for modifying and manipulating the regimes of cavity QED and nanoscale light-matter interactions using a single QD within a planar PC nanocavity has been proposed. By obtaining the analytic GFT of the complex surrounding environment, we have demonstrated the profound effect that an integrated waveguide and an external cavity can have on the transmission of light as well as on the renormalized susceptibility of a single-cavity embedded QD. Besides enhancing the regimes of the usual cavity QED, more generally with the ability to add in more than 1 QD (enabling intercavity QD coupling or intracavity QD coupling [21]) this scheme opens up a possible way to entangle photons and excitons over macroscopic distances.

This work was supported by the Natural Sciences and Engineering Research Council of Canada, and the Canadian Foundation for Innovation.

*Electronic address: shughes@physics.queensu.ca

[1] E. M. Purcell, Phys. Rev. **69**, 37 (1946).
[2] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).
[3] S. John, Phys. Rev. Lett. **58**, 2486 (1987).
[4] P. Lodahl *et al.*, Nature (London) **430**, 654 (2004).
[5] T. Yoshie *et al.*, Nature (London) **432**, 200 (2004).
[6] E. Peter *et al.*, Phys. Rev. Lett. **95**, 067401 (2005).
[7] P. Reitzenstein *et al.*, Nature (London) **432**, 197 (2004).
[8] Y. Akahane *et al.*, Nature (London) **425**, 944 (2003).
[9] A. Badolato *et al.*, Science **308**, 1158 (2005).
[10] W. Yao *et al.*, Phys. Rev. Lett. **95**, 030504 (2005).
[11] A. Rahmani and G. W. Bryant, Phys. Rev. A **65**, 033817 (2002).
[12] J. M. Wylie and J. E. Sipe, Phys. Rev. A **32**, 2030 (1985).
[13] S. Hughes *et al.*, Phys. Rev. Lett. **94**, 033903 (2005).
[14] H. T. Dung, L. Knöll, and D.-G. Welsch, Phys. Rev. A **66**, 063810 (2002).
[15] M. Wubs, L. G. Suttorp, and A. Lagendijk, Phys. Rev. A **70**, 053823 (2004).
[16] A. Cowan and Jeff. E. Young, Phys. Rev. E **68**, 046606 (2003).
[17] S. Hughes and H. Kamada, Phys. Rev. B **70**, 195313 (2004).
[18] E. Kuramochi *et al.*, Phys. Rev. B **72**, 161318(R) (2005).
[19] W. Langbein *et al.*, Phys. Rev. B **70**, 033301 (2004).
[20] E. Waks and J. Vuckovic, Phys. Rev. Lett. **96**, 153601 (2006).
[21] S. Hughes, Phys. Rev. Lett. **94**, 227402 (2005).