

# Single quantum dots for slow and fast light in a planar photonic crystal

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We theoretically investigate slow and fast light propagation and pulse velocity control in a nanocavity containing a single quantum dot side-coupled to a planar-photonic-crystal waveguide. We demonstrate that low coupling strength (i.e., the weak coupling regime) between a cavity and a dot, under on-resonance condition, can lead to delays of about +90 ps for a pulse  $1/e$ -width of 280 ps. The group delay dependence on the various coupling parameters suggests achievable delays of +300 ps and consequently very slow light speeds of around 5000 m/s in a  $1.5 \mu\text{m}$  cavity-waveguide section. We also show that under off-resonant condition one can achieve significant pulse advancement of  $-60$  ps. © 2007 Optical Society of America  
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Slow and fast light in various dispersive media has recently become an active area of research in optics.<sup>1</sup> This interest is motivated by both fundamental light-matter studies and potential applications in optical communication/signal processing and quantum information science. Over the past few years, there have been a number of techniques realized to reduce the velocity of light in various media from atomic to solid-state systems and operating at ultracold to room temperatures. Well-known methods for producing *slow light* include electromagnetically induced transparency,<sup>2</sup> coherent population oscillation,<sup>3</sup> stimulated Raman scattering,<sup>4</sup> and stimulated Brillouin scattering.<sup>5</sup> These systems possess steep normal dispersion over a narrow spectral range for the probe, which can be modified by changing the intensity of another resonant beam called the pump. Another scheme for slow light is the coupled resonator optical waveguides caused by structural (or geometric) dispersion.<sup>6</sup> The latter structures are particularly promising in the sense that they could offer a large group delay-bandwidth product, submillimeter compactness, tunability, and reasonable throughput.<sup>7</sup>

Planar photonic crystals (PPCs) have come a long way in terms of applications to quantum and nonlinear optics driven by technological advances made in the fabricating such structures. Waveguide modes formed by linear defects in PPCs have recently demonstrated low losses.<sup>8,9</sup> Also, the current nanocavities formed by spatial defects in PPCs possess enormous quality factors ( $Q > 10^5$ ) and low effective mode volume ( $V \sim 0.05 \mu\text{m}^3$ ), leading to extremely large  $Q/V$  ratios.<sup>10</sup> Such large ratios are important for studying fundamental cavity QED and for future applications in quantum information science using compact solid-state systems.

Here we investigate slow and fast light in a system that has both material and structural dispersion, namely, a PPC nanocavity containing a single quantum dot (QD) side-coupled to a waveguide in transmission; a schematic of our proposed slow light propagation scheme is shown in Fig. 1. Superluminal and subluminal propagation have been studied experimentally in sophisticated systems with a high-finesse Fabry-Perot (FP) cavity containing ultracold

resonant atoms<sup>11</sup> and also theoretically in a low-finesse FP cavity with silver mirrors containing atomic absorbers.<sup>12</sup> Planar photonic crystals are a natural choice to study slow light compared with atomic system/FP cavities because of their compactness and integrability in realizing all-optical chips. Moreover, photonic crystals as well as other properties in the atomic systems are fixed and cannot be tuned to control the light propagation. Conversely, photonic crystals offer many degrees of freedom, such as the possibility to tune resonant cavity frequencies and produce massive  $Q/V$  ratios at wavelengths compatible with telecoms. Remarkably, it is also possible to deterministically couple single QDs to PPC cavities, both spatially and spectrally.<sup>13</sup>

The coupled resonator optical waveguide structures can be realized in (a) directly coupled resonant structures where the coupling is mediated by evanescent field overlap between adjacent resonators and (b) indirectly coupled resonant structures where coupling is mediated by a field overlap between a resonator and a common waveguide. Slow group velocity of light has been also reported in a 2D array of coupled optical resonators using PPCs.<sup>14</sup> Here we

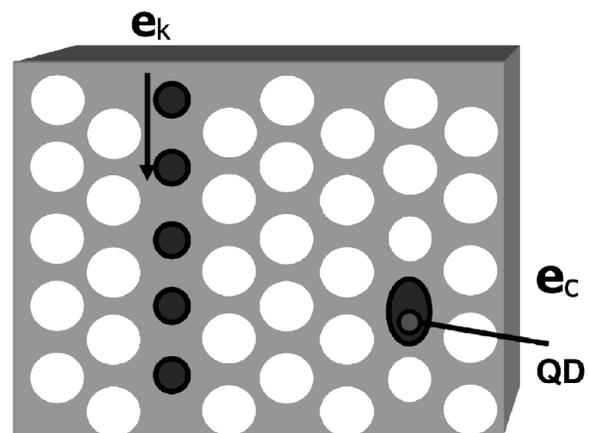


Fig. 1. Schematic of the waveguide/nanocavity coupling scheme using a planar photonic crystal. The quantum dot (QD) is placed near the field antinode location of the nanocavity. The propagating field excites the defect cavity  $e_c$ , which couples to the propagating Bloch mode  $e_k$  and to the embedded QD.

consider a waveguide side-coupled to a nanocavity with a QD using PPCs to study light propagation. Previously, Hughes and Kamada developed a photon Green function formalism<sup>15</sup> to investigate the strong coupling regime in a PPC nanocavity containing a QD side-coupled to a waveguide and subsequently demonstrated vacuum Rabi splitting in reflection (or transmission), although spectral splitting in transmission/reflection is not necessarily a signature of strong coupling. To study pulse propagation in such systems, here we exploit that formalism to derive the complex transmission and use it to calculate the group delay (or the phase time) as a function of cavity detuning. We also verify the group delay predictions, for specific cases, by explicitly calculating profiles of the transmitted pulse and the reference pulses for a carrier tuned near the system resonance. A comparison of the pulse shapes and shifts in their peaks reveals that the transmitted pulse is delayed/advanced by the phase time value at the carrier frequency. We also investigate how the group delay depends on the coupling strength between a nanocavity and a QD and also on the waveguide.

The expression for complex transmission coefficient for a PPC waveguide coupled to cavity and a QD derived using a generalized Green function formalism and Maxwell's equations is given by<sup>15</sup>

$$T(\omega) = 1 + \frac{i\omega\Gamma_c}{\omega_c^2 - \omega^2 - i\omega(\Gamma_c + \Gamma_0) - \omega_c\Sigma}, \quad (1)$$

where the self-energy  $\Sigma = \Omega^2/(\omega_d - \omega - i\gamma/2)$  with  $\Omega^2 = \omega_c|d|^2/[V2\epsilon_0\epsilon(r_d)\hbar]$  for a QD at the cavity-field antinode,  $\epsilon(r_d)$  is the relative electric permittivity in which the QD is embedded ( $r_d$  is the QD spatial position), and  $d$  is the optical dipole moment of the QD electron-hole pair. The other parameters are defined as follows:  $\Gamma_0$  is mode decay rate to account for intrinsic out-of-plane scattering,  $\Gamma_c$  is the coupling rate between the nanocavity and waveguide (its explicit expression depends on the mode overlap and scaled inversely with group velocity<sup>15</sup>),  $\omega_c$  is the characteristic resonant frequency of a nanocavity,  $\gamma$  is the phenomenological nonradiative decay rate of a QD, and  $\omega_d$  is the resonant frequency of a dot. Note that  $\Omega$  is a measure of the coupling strength between the QD and the cavity. In our calculations we first choose the QD to be resonant with the cavity ( $\omega_c = \omega_d$ ). The group delay for the transmitted pulse is<sup>12,16</sup>  $\tau_T = \partial\phi_T/\partial\omega|_{\omega=\omega_p}$ , where  $\phi_T$  is the phase of complex transmission coefficient in Eq. (1). We also assume an input Gaussian pulse launched into the waveguide given by  $A_I(t) = e^{-(t^2/\tau^2)}e^{-i\omega_p t}$ , where  $\tau$  is  $1/e$  pulse width and  $\omega_p$  is the pulse carrier frequency, and the transmitted pulse shape is calculated from

$$A_T(t) = \int_{-\infty}^{\infty} d\omega T(\omega)A_I(\omega)e^{-i\omega t}, \quad (2)$$

with  $A_I(\omega)$  the input pulse spectrum.

For a close to optimal choice of parameters, Fig. 2(a) displays the transmission in the waveguide ver-

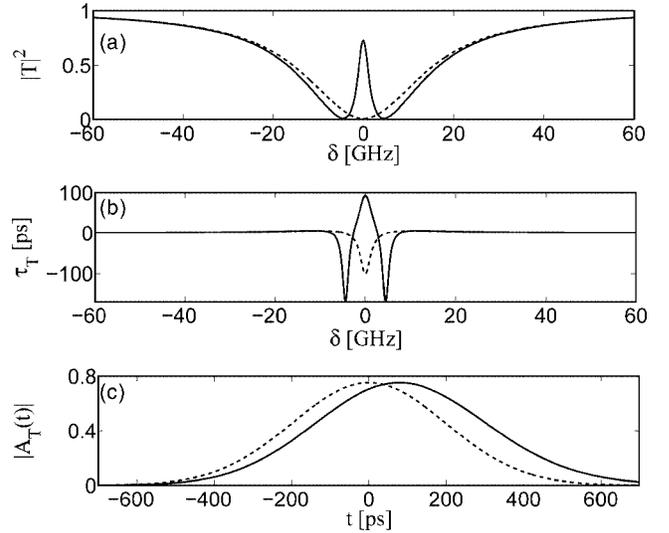


Fig. 2. (a) Intensity transmission  $|T|^2$  of a PPC waveguide coupled to a nanocavity system with the QD (solid curve) and without the QD (dashed curve) as a function of cavity detuning [ $\delta = (\omega - \omega_c)/2\pi$ ]. The parameters are as follows:  $\Gamma_c/2\pi = 28.8$  GHz ( $\hbar\Gamma_c \approx 0.12$  meV),  $\Gamma_0/2\pi = 2.9$  GHz,  $\gamma/2\pi = 0.5$  GHz,  $\Omega/2\pi = 6.3$  GHz. (b) Corresponding group delay  $\tau_T$ . (c) Transmitted pulse shape (solid curve) compared with the reference pulse (dashed curve normalized to the peak of the transmitted pulse) for an input Gaussian pulse with width  $\tau = 280$  ps and carrier chosen at cavity resonance.

sus cavity detuning  $\delta$ . The transmission increases rapidly in the presence of QDs for frequencies lying near the cavity resonance. This modified resonance structure results in substantial positive delay, which is shown in the group delay (or phase time) plot of Fig. 2(b). In Fig. 2(c) we also display how the group delay affects the pulse shapes, showing a peak shift in time of nearly +80 ps compared with the input (reference) pulse.

We remark that to optimally realize slow light propagation in the waveguide operation in a weak coupling regime or at the onset of vacuum Rabi splitting is required. Here the dipole coupling to the cavity-waveguide system leads to the transparency in an otherwise reflective system. This effect is somewhat similar to the phenomenon observed in an EIT medium<sup>2</sup> that has been used to slow down light. A related transparency effect has been noted by Waks and Vuckovic<sup>17</sup> using a two waveguide-cavity system with a QD.

Next we investigate how the group delay and transmission characteristics depend on the various coupling rates of the system. From Eq. (1) and the group delay expression it follows that, at cavity resonance, the absolute value of transmission coefficient and the corresponding group delay are given by

$$|T| = 1 - \frac{\Gamma_c\gamma}{2\Omega^2 + (\Gamma_c + \Gamma_0)\gamma}, \quad (3)$$

$$\tau_T = \frac{(2\Omega^2 - \gamma^2)\Gamma_c}{[2\Omega^2 + \gamma\Gamma_0][2\Omega^2 + \gamma(\Gamma_0 + \Gamma_c)]}. \quad (4)$$

Figure 3(a) shows the group delay time as function of cavity-QD coupling strength, while 3(b) shows the in-

fluence of the cavity-waveguide coupling strength. As can be recognized, smaller values of  $\Omega$  lead to large group delays, but then the transparency width becomes narrower and consequently the peak value reduces. A similar feature can be noted for large values of  $\Gamma_c$ . In this structure with more fine tuning and optimal choice of the parameters we predict that one can reasonably expect to go up to group delays of more than 300 ps. For a device (waveguide-cavity) section of  $1.5 \mu\text{m}$ , the light speed becomes only 5000 m/s at a group delay of +300 ps.

Finally, we briefly comment that in the case when a cavity and a dot are nonresonant ( $\omega_c \neq \omega_0$ ), it is possible to achieve +80 ps of group delay even up to 2 GHz off-resonance with a carrier tuned at cavity resonance. Another interesting thing to be noted is that when they are off-resonance by 10 GHz one can achieve pulse advancement (or negative delay<sup>3,12,18</sup>) of -60 ps (see Fig. 4) with at least 13% peak ampli-

tude in the transmitted pulse with minimal distortion. Last, but not least, we highlight that the waveguide mode can be operative in a low-loss propagation regime, away from the photonic band-edge, which is important since light propagation using slow propagation modes leads to substantial disorder-induced scattering loss.<sup>8,19</sup> In principle, the group delay in such structures can also be tuned dynamically going from off- to on-resonance condition, for example, by applying an external field (optical<sup>20</sup> or a bias<sup>21</sup>) to cause a Stark shift of the QD exciton resonance.

In conclusion, we have introduced a novel slow, and also fast, light scheme that uses integrated PPCs, with a propagating waveguide mode coupled to a nanocavity containing a single QD.

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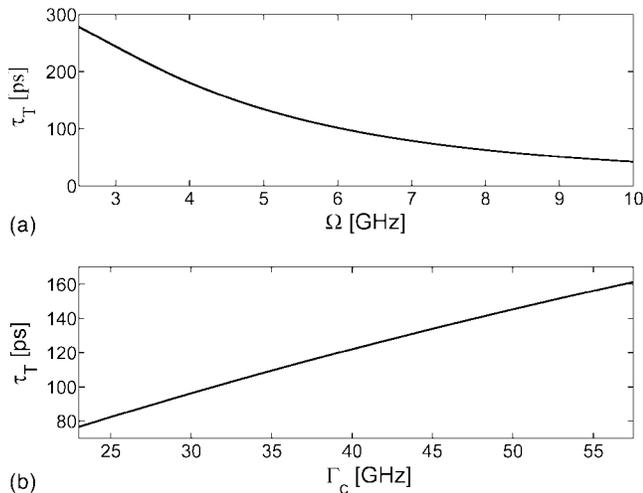


Fig. 3. Group delay  $\tau_T$  dependence on the coupling parameters using a pulse carrier tuned at cavity resonance (a) versus cavity-dot coupling  $\Omega$ , (b) versus cavity waveguide coupling  $\Gamma_c$ . The other parameters are the same as in Fig. 2.

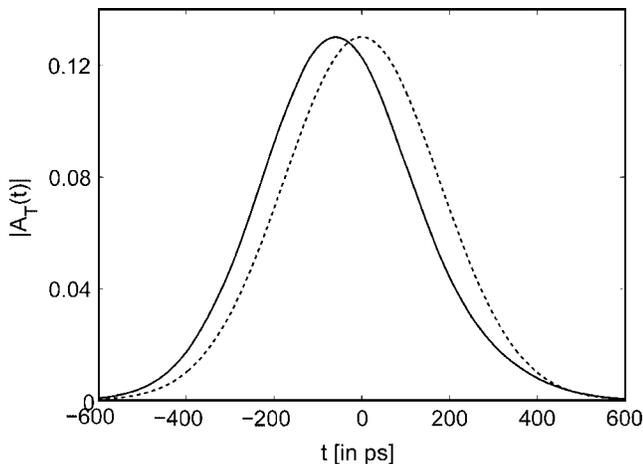


Fig. 4. Advancement in the transmitted pulse shape (solid curve) compared with the reference pulse (dashed curve) with width  $\tau = 250$  ps and the carrier tuned near the cavity resonance. The other parameters are the same as in Fig. 2.