Single-quantum-dot strong coupling in a semiconductor photonic crystal nanocavity side coupled to a waveguide

S. Hughes* and H. Kamada
NTT Basic Research Laboratories, NTT Corporation, 3-1 Morinosato-Wakamiya, Atsugi, Kanagawa 243-0198, Japan
(Received 1 June 2004; revised manuscript received 3 August 2004; published 10 November 2004)

We present an intuitive photon Green function formalism to study *strong coupling* via reflection or transmission of light in a planar-photonic-crystal waveguide that is side coupled to a nanocavity containing only one quantum dot. Analytical results allow physical insight and useful formulas to optimally design such integrated systems in a realistic way and we subsequently demonstrate that strong coupling should be achievable using present day photonic crystals and quantum dots, yielding single-exciton vacuum Rabi splittings of around 0.1–1 meV.

DOI: 10.1103/PhysRevB.70.195313 PACS number(s): 78.67.Hc, 41.20.Jb, 42.55.Tv, 42.70.Qs

I. INTRODUCTION

Driven by their fascinating atomiclike charge-carrier energy levels and by their potential in future quantum computing architectures, optical processes in single quantum dots (QD's) are currently receiving a lot of interest within the scientific community.[1] Both theoretical and experimental studies are being undertaken in an attempt to clarify their underlying physical processes with a view to future photonic applications such as single photon- and enhanced light-emission.[2] As a prerequisite for understanding their quantum optics potential, single QD experiments have been undertaken to investigate weak coupling effects, such as modifying their spontaneous emission lifetimes in a micropost cavity,[3,4] which yield Purcell factors of around 5. While enhanced coupling in piller semiconductor microcavities has been theoretically investigated,[5] the resulting quality factors (Q's) and inverse mode volumes are hitherto found to be too small to achieve *strong coupling*.[6] To possibly overcome this hurdle, it has been proposed that planar photonic crystals (PPC's) offer several advantages over the piller microcavity system.[7] Moreover, PPC Q's have very recently undergone *order-of-magnitude* improvements, with greater than up 45 000 now realized experimentally.[8]

In this work we present a semiquantitative analytical formalism, with example calculations, for a PPC defect waveguide side-coupled to a nanocavity containing a single QD. This work is motivated partly on fundamental grounds and partly due to tremendous progress in realizing single QD's and miniaturized defect waveguides/microcavies in semiconductor PPC's;[8,9] indeed these material systems are now being coupled together.[10] We also highlight that although cavity QED experiments have also been investigated for semiconductor nanocrystals with dielectric microspheres,[11] which can have enormous quality factors, excessive pure dephasing has so far prevented any signs of strong coupling. Thus there is now a timely need to try to combine and exploit the nanoscale PPC cavity and single QD materials. This study is also motivated by the fact that simple intuitive theoretical formalisms have not been presented for such a system, and the current trend has been to resort to large scale numerical simulations, that too frequently predict unrealistic values as well as offer little physical insight to the problem at hand. With regard to the present investigation of strong coupling, the PPC system is attractive for two main reasons: (i) it allows a simple waveguide approach for excitation and detection of the coupling via a line defect waveguide, and (ii) the miniaturized microcavity can be included in a natural and integrated way. This avoids complicated vertical excitation and detection techniques and enables high-Q and small-volume nanocavities. We remark that vertical excitation schemes with plane waves are somewhat impractical for exciting defect modes in the photonic bandedge and give rise to very complex reflectivities and additional broadening mechanisms; thus we specialize our study to in-plane propagation, which has the large advantage of containing an integrated waveguide.

Our main goal is to obtain useful expressions for experimentalists and theorists alike, to realistically study if strong coupling should be observable with today's QD's and PPC's, and to present model calculations and important scaling rules using representative experimental parameters. To meet this requirement, we apply a photonic Green function tensor (GFT) technique that allows us to derive simple yet valid analytical expressions for the reflection and transmission of light. Real-space GFT's have been applied recently for nonlinear coupling of microcavities in PPC guides,[12] and to study *extrinsic* optical scattering loss due to disorder in PPC waveguides (no nanocavity).[20] In Fig. 1 we show a simple schematic of the nanocavity and PPC waveguide system, with the QD positioned in the nanocavity. Below, we discuss the theoretical formalism, present representive numerical results, and discuss the underlying physical effects.

II. THEORETICAL FORMALISM

We first define the two relevant optical functions, $\Delta\epsilon_w$ and $\Delta\epsilon_c$, that represent the change in the relative electric permittivity by adding a defect waveguide (e.g., a line defect of missing holes) and a cavity, respectively, to the perfect PPC (no waveguide) with ϵ_{pc}. Thus $\epsilon_w=\epsilon_{pc}+\Delta\epsilon_w$, $\epsilon_c=\epsilon_{pc}+\Delta\epsilon_c$, and $\epsilon_t=\epsilon_{pc}+\Delta\epsilon_c+\Delta\epsilon_w$ represent the permittivities of the PPC defect guide, nanocavity, and the total coupled system, respectively. The latter we term the total background system with the understanding that there is no embedded QD yet.

We can write out the solution for the propagating electric field as

$$\mathbf{E}(\mathbf{r};\omega) = \mathbf{E}^B(\mathbf{r};\omega) + \int d\mathbf{r}' \overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}';\omega)\cdot\Delta\epsilon_c(\mathbf{r}')\mathbf{E}^B(\mathbf{r}';\omega), \tag{1}$$

where $\overleftrightarrow{\mathbf{G}}$ is the GFT of the PPC including both the defect cavity and waveguide, and $\mathbf{E}^B(\mathbf{r};\omega)$ is a solution to the PPC plus waveguide only (no nanocavity, with ϵ_w). The GFT is defined from

$$\nabla\times\nabla\times\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}';\omega) - \frac{\omega^2}{c^2}\epsilon_t(\mathbf{r})\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}';\omega) = \frac{\omega^2}{c^2}\overleftrightarrow{\mathbf{1}}\delta(\mathbf{r}-\mathbf{r}') \tag{2}$$

with $\overleftrightarrow{\mathbf{1}}$ the unit tensor. To derive the appropriate GFT for the coupled defect cavity-waveguide system we write the general solution by expanding into the PPC modes

$$\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}';\omega) = \sum_{\alpha,\beta} A_{\alpha,\beta}(\omega)\mathbf{e}_\alpha(\mathbf{r})\otimes\mathbf{e}^*_\beta(\mathbf{r}') \tag{3}$$

with $A_{\alpha,\beta}$ the expansion coefficients that we seek to obtain, $\otimes$ the dyadic multiplier, and $\mathbf{e}_{\alpha/\beta}(\mathbf{r})$ represent the modes of the PPC system at frequency ω that may belong to either the cavity or waveguide; we assume that both these sets of modes are *weakly coupled* to each other as is usually the case for such PPC systems, and is the desired case here.

We take the **E**-field waveguide modes $\mathbf{E}_k = \sqrt{a/L}e^{ikx}\mathbf{e}_k$ with the normalization $\int_{\text{unit cell}} d\mathbf{r}\,\epsilon_w(\mathbf{r})|\mathbf{e}_k(\mathbf{r})|^2 = 1$, carried out over 1 unit cell of the PPC, with L the length of the waveguide and a the periodic pitch in the direction of propagation (x). We also define the fundamental (and dominant) cavity mode normalized from the **E** field of the cavity: $\mathbf{e}_c = \boldsymbol{\phi}_c/\sqrt{V_m}$, with $|\boldsymbol{\phi}_c|^2 = |\mathbf{e}_c|^2/\max[\epsilon_c|\mathbf{e}_c|^2]$, where the effective mode volume $V_m = \int_{\text{all space}} d\mathbf{r}\,\epsilon_c(\mathbf{r})|\boldsymbol{\phi}_c(\mathbf{r})|^2$.

In order to obtain the expansion coefficients of the GFT, we expand the Dirac delta function $\overleftrightarrow{\mathbf{1}}\delta(\mathbf{r}-\mathbf{r}') = \sum_\alpha \epsilon_t(\mathbf{r})\mathbf{e}_\alpha(\mathbf{r})\otimes\mathbf{e}^*_\alpha(\mathbf{r}')$. Utilizing a technique similar to that described by Cowan and Young,[12] it follows that the expansion coefficients can be explicitly obtained from a matrix equation. In working this out we assume that the only contribution to the GFT away from the nanocavity (at $\mathbf{r}\to\pm\infty\hat{x}$) that can contribute to the transmission or reflection of light is from the forward or backward propagating waveguide mode of interest; this is simply the homogeneous solution of the PPC waveguide at $k=h$, where h is the propagation constant at frequency ω. The relevant GFT expansion coefficients thus take the form

$$A_{\alpha,\beta(=k,c)}(\omega) = \frac{\sum_k \dfrac{\omega^2\omega_k^2\sqrt{\dfrac{a}{L}}\displaystyle\int d\mathbf{r}'\mathbf{e}_k(\mathbf{r}')e^{ikx'}\Delta\epsilon_c(\mathbf{r}')\mathbf{e}^*_c(\mathbf{r}')}{\omega_k^2-\omega^2}}{\omega_c^2-\omega^2-\dfrac{ia\omega^3}{2v_g}\left|\displaystyle\int d\mathbf{r}'\Delta\epsilon_c(\mathbf{r}')\mathbf{e}^*_c(\mathbf{r}')\mathbf{e}_h(\mathbf{r}')e^{ihx'}\right|^2} \tag{4}$$

with v_g the group velocity of the propagating mode and ω_c is the cavity resonance frequency. The symmetry of the defect mode is such that the overlap integrals involving h and $-h$ are assumed identical. We note that the $\Delta\epsilon_c$ term appears naturally in the expansion coefficient as it cannot exploit the orthogonality condition that was used to obtain the $\omega_\alpha^2-\omega^2$ terms in the equation above (see, also, Ref. 12). Furthermore, the expansion coefficient within the center of the nanocavity ($A_{c,c}$) can be obtained by simply replacing the numerator in Eq. (4) by ω^2; this will be used later to obtain the peak electric field required for strong coupling.

We now label the forward and backward homogeneous solutions as $\mathbf{E}_+(\mathbf{r}) = \sqrt{a/L}\mathbf{e}_h(\mathbf{r})e^{ihx}$ and $\mathbf{E}_-(\mathbf{r}) = \mathbf{E}^*_+(\mathbf{r})$, respectively. Subsequently, we can derive the propagating field in the waveguide as

$$\begin{aligned}\mathbf{E}(\mathbf{r};\omega) = \mathbf{E}^B(\mathbf{r};\omega) &+ \frac{\Theta(x)\mathbf{e}_h(\mathbf{r})e^{ihx}\dfrac{ia\omega^3}{2v_g}\displaystyle\int d\mathbf{r}'\Delta\epsilon_c(\mathbf{r}')\mathbf{e}^*_c(\mathbf{r}')\mathbf{E}_+(\mathbf{r}')\int d\mathbf{r}'\Delta\epsilon_c(\mathbf{r}')\mathbf{e}_c(\mathbf{r}')\mathbf{e}^*_h(\mathbf{r}')e^{-ihx'}}{\omega_c^2-\omega^2-\dfrac{ia\omega^3}{2v_g}\left|\displaystyle\int d\mathbf{r}'\Delta\epsilon_c(\mathbf{r}')\mathbf{e}^*_c(\mathbf{r}')\mathbf{e}_h(\mathbf{r}')e^{ihx'}\right|^2}\\ &+ \frac{\Theta(-x)\mathbf{e}^*_h(\mathbf{r})e^{-ihx}\dfrac{ia\omega^3}{2v_g}\displaystyle\int d\mathbf{r}'\Delta\epsilon_c(\mathbf{r}')\mathbf{e}^*_c(\mathbf{r}')\mathbf{E}_-(\mathbf{r}')\int d\mathbf{r}'\Delta\epsilon_c(\mathbf{r}')\mathbf{e}_c(\mathbf{r}')\mathbf{e}^*_h(\mathbf{r}')e^{-ihx'}}{\omega_c^2-\omega^2-\dfrac{ia\omega^3}{2v_g}\left|\displaystyle\int d\mathbf{r}'\Delta\epsilon_c(\mathbf{r}')\mathbf{e}^*_c(\mathbf{r}')\mathbf{e}_h(\mathbf{r}')e^{ihx'}\right|^2},\end{aligned} \tag{5}$$

where we have assumed that the integral over the waveguide Bloch functions ($\mathbf{e}_k$) vary slowly with respect to k in carrying out the complex k integrations. It is also clear that the Heaviside step functions $\Theta(x)$ and $\Theta(-x)$ correspond to the solutions for the transmitted and reflected electric fields, and we can take the following input field to the waveguide

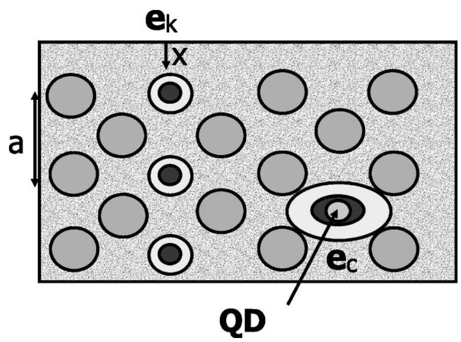

FIG. 1. Schematic of the proposed PC waveguide/nanocavity coupling scheme, showing a plan view. The quantum dot (QD) is located within the nanocavity which has an exciton resonance close to the cavity resonance. The propagating field excites the defect cavity $\mathbf{e}_c$, which couples to the propagating Bloch mode $\mathbf{e}_k$ and to the embedded exciton. The modes are shown by the concentric circles. The pitch in the direction of propagation (x direction) is a, and the grey circles represent the air holes in a semiconductor photonic crystal stucture.

$\mathbf{E}^B(\mathbf{r}\rightarrow-\infty)=\mathbf{E}_+(\mathbf{r})$, that is injected far from the nanocavity. Focusing on the solution for the reflection of light, we can then write the reflectivity as $r(\omega)=i\omega\Gamma_c(\omega)/[\omega_c^2-\omega^2-i\omega\Gamma_c(\omega)]$, where the cavity linewidth rate $\Gamma_c(\omega)=\omega^2 a/(2v_g)|\int_{\text{all space}} d\mathbf{r}\Delta\epsilon_c(r)\exp(ihx)\mathbf{e}_h(\mathbf{r})\mathbf{e}_c^*(\mathbf{r})|^2$, which is obtained from Eq. (5). We can thus define $Q=\omega_c/\Gamma_c$, that is consistent with PPC experimental measurements of side-coupled cavities.

Our final task is to add in the QD self-consistently. To do this we exploit the Dyson equation that can be written, in operator form, as $\mathbf{G}^d=\mathbf{G}+\mathbf{G}\cdot\Delta\varepsilon_d\cdot\mathbf{G}^d$, where $\mathbf{G}^d$ now includes the response of the QD. We assume a single QD with a volume V_d and diameter much smaller than the wavelength of light; thus the QD permittivity $\Delta\varepsilon_d(r_d)=|d|^2/[2V_d\varepsilon_0\hbar(\omega_d-\omega-i\Gamma_d)]$, with $d=p_{cv}\int d\mathbf{r}\langle\Phi_{1s}|\psi_e^\dagger(\mathbf{r})\phi_h^\dagger(\mathbf{r})|0\rangle$ the exciton dipole moment (p_{cv} is the interband matrix element, $\phi_{e/h}^\dagger$ the creation operators of the corresponding electron/hole, and Φ_{1s} is the 1s exciton wave function[13]), Γ_d the nonradiative decay time of the QD, and ω_d is the QD resonance frequency. We have assumed the fundamental 1s exciton resonance only and employed the rotating-wave approximation for the QD permittivity. Subsequently we can obtain the reflectivity in the presence of the QD. We have

$$r(\omega)=\frac{i\omega\Gamma_c(\omega)}{\omega_c^2-\omega^2-i\omega\Gamma_c(\omega)-\omega\Sigma(\omega)}, \qquad (6)$$

where $\omega\Sigma(\omega)=V_d\mathbf{G}(r_d,r_d)\Delta\varepsilon_d(r_d)(\omega_c^2-\omega^2-i\omega\Gamma_c)$ is the radiation self-energy contribution. Since the QD is located in the weakly coupled nanocavity with a large Q, the dominant contribution to the GFT is from the cavity mode coupled to itself ($A_{c,c}$ is the relevant GFT expansion coefficient) and $\mathbf{G}(r_d,r_d)=\omega^2|e_c(r_d)|^2/(\omega_c^2-\omega^2-i\omega\Gamma_c)$ is the only term needed for the Dyson equation. Positioning the QD at the field antinode in the nanocavity and using the normalized cavity mode field, we have $\Sigma(\omega)=\Omega^2/(\omega_d-\omega-i\Gamma_d)$, where $\Omega^2=\omega|d|^2/(2\varepsilon_0\varepsilon_d\hbar V_m)$ with ε_d is the relative permittivity of the QD. Equation (6) covers both weak and strong coupling regimes and is rigorously valid within our stated approximations and model PPC system.

The present formalism enables one to design a particular PPC waveguide and nanocavity system, while including the QD parameters in a natural way. It is valid for weak coupling between the waveguide and nanocavity and a cavity resonance within the photonic band gap, which is the usual case for these materials. The most important physical processes can be optimized independently in the separate systems, or, alternatively, the most important parameters can be obtained from experiments. The essential point is that the above facilitates their coupling and side-steps the problem of blindly running large scale numerical simulations. The QD self-energy is incorporated through a self-consistent Dyson equation that includes the PPC GFT's and the QD permittivity from quantum theory; the GFT at the QD location is assumed to be dominated by the cavity mode that is well justified for our high Q and weakly coupled cavity.

In addition, if one wishes to probe the time dependent dynamics, the above formulas can be Fourier transformed. For example, assuming a delta function incident pulse and employing the rotating-wave approximation, then $r(t)=\cos(\Omega t)\exp(-i\omega_c t)\exp(-\Gamma t)\Theta(t)$, where we have assumed $\Gamma=(\Gamma_c+\Gamma_d)/2\ll\Omega$. Additionally, one can also show that the time dependent polarisation in the QD $P(t)=A/\Omega\sin(\Omega t)\exp(-i\omega_c t)\exp(-\Gamma t)\Theta(t)$, where A is proportional to the the electric field at the QD location. As expected, the polarization is in quadrature with $r(t)$, and similar results have been obtained for 1D semiconductor microcavities.[14]

III. NUMERICAL RESULTS

Instead of carrying out a detailed numerical design sweep of cavity Q's and PPC modes (which, of course, is certainly possible), we employ the following *experimentally achievable parameters*: $Q=10\,000$, $\omega_c=\omega_d$ (cavity detuning is investigated below), $\omega_c/(2\pi)\approx 230$ THz, $\Gamma_c(\omega_c)=0.1$ meV, $\Gamma_d(\ll\Gamma_c)=0.002$ meV and higher, and extract the exciton dipole moment from single QD experiments[15] ($d=60$ Debye). We note that larger Q's are certainly possible but we will use reasonable values in the presence of the waveguide. In the regime of strong coupling we expect vacuum Rabi splitting.[6] In Figs. 2(a)–2(c) we show the reflection, $R=|r|^2$, for both on (solid) and off (dashed) resonance with the QD exciton for three different coupling strengths, using a nominal $\Gamma_d=0.002$ meV consistent with experiments[17] (the influence of dephasing is investigated below). In (a) the mode volume is 0.05 μm^3, while in (b)/(c), the ratio of d^2/V_m has been increased/decreased by a factor of 10. The fact that all three excitation scenarios result in clear vacuum Rabi splitting, with a range 0.1–2 meV, means that the scheme should also be robust with respect to manufacturing imperfection; though this is partly accounted for already by using experimentally obtainable parameters.

To investigate the influence of detuning, in Fig. 3 we show the nominal reflection results corresponding to Fig. 2(a) but with a cavity detuning $(\omega_d-\omega_c)$ of (a) 0.3 meV, (b)

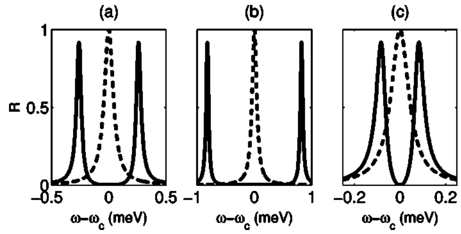

FIG. 2. Reflection versus frequency for on (solid curce) and off (dashed curve) resonance with the quantum dot for three different effective mode volumes (see text). (a) Nominal parameters as given in the text. (b) The ratio of d^2/V_m increased by a factor of 10. (c) The ratio of d^2/V_m decreased by a factor of 10.

0.6 meV, and (c) 0.9 meV. Physically one would tune the QD to the cavity, for example, by changing the temperature. These results demonstrate that the Rabi splittings are robust for cavity detunings of up to several meV. However, the secondary sharp peaks (at the positive detunings) are strongly influenced by the amount of dephasing; numerically we have verified however that all the above cases survive even with an order of magnitude increase in the dephasing rate, while larger detuning of greater than 1 meV are barely detectable with the larger dephasing rate.

Next, we study the role of exciton dephasing. In Fig. 4, we show the on-resonance nominal reflection simulations with (a) nominal case (0.002 meV), (b) 10 times larger dephasing rate (0.02 meV), and (c) 100 times larger dephasing rate (0.2 meV). As expected, the influence of dephasing is to adjust the height of the Rabi sidebands, which eventually broaden out and may disappear for very large dephasing. The nominal Rabi splitting will tolerate up to a maximum Γ_d of around 0.3 meV (comparable to the splitting energy) for the QD. We remark that our dephasing rates are actually quite modest with what one measures with today's high quality single QD's at low temperature, which can even be dominated with only radiative decay for homogeneous materials;[18] typical measured low temperature single QD dephasing rates fall somewhere between cases (a) and (b).

IV. CLASSICAL OR QUANTUM OPTICS EFFECT?

So where is the quantum optics? We highlight that all the observed features are a result from linear-dispersion theory specialized to the PPC system and a single QD. It has been known for quite some time now that Rabi splitting can appear as a feature of linear dispersion.[16] By employing a quantum theory the predicted Rabi splitting is expected to be $V=2\sqrt{g}$, where g is the QD-cavity coupling constant defined from $g\hbar=|\langle \mathbf{d}\cdot\mathbf{E}\rangle|$, with **d** and **E** the dipole and **E** operators;[5] the quantized electric field can then be expressed in terms of the nanocavity mode used above. One can then write $V=2\sqrt{\omega_c d^2/(2\epsilon_0\epsilon_d\hbar V_m)}$, and this yields about $V=0.3$ meV for the nominal results obtained in Fig. 2(a). This splitting is consistent identical to the calculations above, but of course we do not expect it to be exactly the same because of scattering processes (neglected in the quantum dressed state picture). It is also possible to obtain *analytically predicted* Rabi splitting from our derived formula above [Eq. (6)]. Assuming $A\ll\omega_c$ and neglecting Γ_d as in the quantum case, then the GFT theory predicts $V_{PC}=\sqrt{4\Omega^2}$ which is, not suprisingly, *identical* to the predicted splitting from quantum theory. Thus we have indeed introduced a valid theoretical formalism to see signatures of vacuum Rabi splitting.

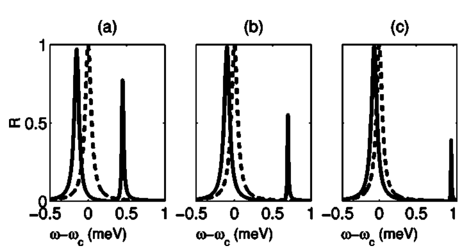

FIG. 3. Reflection versus frequency corresponding to Fig. 2(a) but with a near-resonance (solid curve) cavity detuning $(\omega_d-\omega_c)$ of (a) 0.3 meV, (b) 0.6 meV, and (c) 0.9 meV.

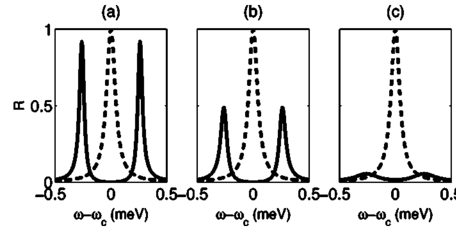

FIG. 4. Reflection versus frequency corresponding to Fig. 2(a) with (a) nominal dephasing rate (0.002 meV; see text), (b) 10 times larger dephasing rate, and (c) 100 times larger dephasing rate.

The important difference between the present study and the quantum well exciton polariton case, or indeed any other collective solid state system, is in how the system behaves when an extra photon is added into the cavity. Classical behavior exhibits essentially no change, since the splitting relies on a large $\sqrt{n}$ with n the number of excitons;[19] however, a large number of photons may result in saturation behavior and possibly optical bistability. In contrast, in the present case, one may expect the formation of the so-called Jaynes-Cummings ladder where quantum jumps occur in the Rabi splitting; this latter phenomenon will of course depend strongly on the selection rules of the QD and the allowed number of single excitons. Finally, the fact that the present 1 exciton study does not exhibit *uniquely quantum* Rabi splitting is, of course, not too surprising as one needs more than 1 photon to probe any quantum statistics. Future work will report on dynamical photon entanglement effects between 2 QD's using again GFT's, but within a quantum optics formalism.

V. ROLE OF MANUFACTURING IMPERFECTIONS, CAVITY LOSS, AND NONLINEAR INTERACTIONS

Before closing, a few words about manufacturing imperfections/disorder, loss, and nonlinear optics are in order. Our formalism above can also include these effects. For the present investigation, this was partly not necessary because of the experimentally employed parameters that we used in the first place, and our main focus here was restricted to the linear optics regime. To add in cavity loss, a Γ_0 can imply be added to the Γ_c appearing in the demoninator of Eq. (6), with a net consequence that the linewidths are more broad and the reflectivities are reduced. Physically, Γ_0 comes from a finite overlap with the radiation modes of the PPC, and for a good cavity it will have little qualitative effects on the findings above. Nonlinear coupling to the cavity (without a QD) has been investigated already,[12] though an extension to add in QD nonlinearities in the present framework is left to future investigations. To include manufacturing imperfections, two basic modifications are needed above. First the resulting GFT for the mixed system can be expanded iteratively and random fluctuations can be added in to the permittivities ϵ_w and ϵ_c, and explicit expressions can be derived within a second-order Born approximation. This results in backscatter, forward-scatter, and out-of-plane scatter, even in the absence of a coupled cavity. Such a formalism has very recently been successfully employed to explain extrinsic scattering loss in PPC waveguides.[20] In operator notation, this suggests that $\mathbf{G}^t = \mathbf{G} + \mathbf{G}\cdot\Delta\epsilon_r\cdot\mathbf{G}$, where $\Delta\epsilon_r$ ($\langle\Delta\epsilon_r\rangle = 0$) is the random change in permittivity due to disorder or roughness interface fluctuations, $\mathbf{G}$ is the GFT that we have derived above, and $\mathbf{G}^t$ is the true GFT that includes disorder self-consistently. For the second modification that is required to include imperfections, the employed permittivity for the cavity linewidth rate would include a $\Delta\epsilon_r$ term as well thus modifying the resulting $Q's$ and overlap integrals that are needed to calculate the reflectivity. This makes our GFT formalism all the more powerful over pure numerical efforts that only consider idealized material systems.

VI. CONCLUSIONS

To conclude, we have derived semiquantitative analytical formulas, using a photonic GFT formalism with a self-consistent Dyson equation, to allow an investigation of strong coupling in an integrated coupled PPC slab waveguide side-coupled to a nanocavity containing 1 QD. Our results imply that single-exciton strong coupling should be observable within today's state-of-the-art QD's and PPC's, producing Rabi splittings of around 0.3 meV in weakly coupled nanocavities with a Q of about 10 000 that include 1 QD. The present formalism can also be naturally extended to include nonlinear optical processes, both within the cavity and within the QD. These results have important implications for utilizing quantum optical processes in nanocavity-influenced single QD's, and future work will address multi-exciton coherence and *unique* quantum optical effects.

Note added. Single-quantum-dot strong coupling has recently been observed experimentally in a PPC nanocavity,[21] based on the design in Ref. 8.

ACKNOWLEDGMENTS

Discussions with D.S. Citrin, L. Ramunno, and M. Notomi are gratefully acknowledged.

*Electronic address: hughes@will.ntt.brl.co.jp

[1] See, for example, *Quantum Coherence, Correlation and Decoherence in Semiconductor Nanostructures*, edited by T. Takagahara (Academic, New York, 2003).

[2] M. Pelton, C. Santori, J. Vukovi, B. Zhang, G. S. Solomon, J. Plant, and Y. Yamamoto, Phys. Rev. Lett. **89**, 233602 (2002).

[3] J. Vuckovic, D. Fattal, C. Santori, G.J. Solomon, and Y. Yamamoto, Appl. Phys. Lett. **82**, 3596 (2003).

[4] G. S. Solomon, M. Pelton, and Y. Yamamoto, Phys. Rev. Lett. **86**, 3903 (2001).

[5] L. C. Andreani, G. Panzarini, and J-M. Gerard, Phys. Rev. B **60**, 13276 (1999).

[6] For a textbook discussion, see, for example, P. Meystre and M. Sargent III, *Elements of Quantum Optics*, 3rd ed. (Springer, Berlin, 1997).

[7] See, for example, J. Vuckovic and Y. Yamamoto, Appl. Phys. Lett. **82**, 2374 (2003).

[8] Y. Akahane, T. Asano, B.S. Song, and S. Noda, Nature (London) **425**, 944 (2003).

[9] See, for example, M. Notomi, K. Yamada, A. Shinya, J. Takahashi, C. Takahashi, and I. Yokohama, Phys. Rev. Lett. **87**, 253902 (2001); H.Y. Ryu, S.H. Kwon, Y.J. Lee, Y.H. Lee, and J.S. Kim, Appl. Phys. Lett. **80**, 3476 (2002).

[10] See, for example, T. D. Happ, I.I. Tartakovskii, V.D. Kulakovskii, J.-P. Reithmaier, M. Kemp, and A. Forchel, Phys. Rev. B **66**, 041303(R) (2002); K. Hennessy, C. Reese, A. Badolato, C.F. Wang, A. Imamoglu, P.M. Petroff, and E. Hu, Appl. Phys. Lett. **83**, 3650 (2003).

[11] H. Wang, in *Quantum Coherence, Correlation and Decoherence in Semiconductor Nanostructures* (Ref. 1), Chap. 8.

[12] A. R. Cowan and J.F. Young, Phys. Rev. E **68**, 046605 (2003).

[13] Y.Z. Hu *et al.*, Phys. Rev. B **42**, 1713 (1990).

[14] D.S. Citrin, Phys. Rev. Lett. **77**, 4596 (1996).

[15] H. Kamada, H. Gotoh, J. Temmyo, T. Takagahara, and H. Ando, Phys. Rev. Lett. **87**, 246401 (2001).

[16] Y. Zhu, D.J. Gauthier, S.E. Morin, Q. Wu, H.J. Carmichael, and T.W. Mossberg, Phys. Rev. Lett. **64**, 2499 (1990).

[17] P. Borri, W. Langbein, S. Schneider, U. Woggon, R.L. Sellin, D. Ouyang, and D. Bimberg, Phys. Rev. Lett. **87**, 157401 (2001).

[18] W. Langbein, P. Borri, U. Woggon, V. Stavarache, D. Reuter, and A. D. Wieck, Phys. Rev. B **70**, 033301 (2004).

[19] See, for example, V. Savona, L. C. Andreani, P. Schwendimann, and A. Quattropani, Solid State Commun. **93**, 733 (1995).

[20] S. Hughes, L. Ramunno, J.F. Young, and J.E. Sipe (unpublished).

[21] T. Yoshie, A. Scherer, J. Hendrickson, G. Khitrova, H.M. Gibbs, G. Rupper, C. Ell, O.B. Shchekin, and D.G. Deppe (unpublished).