Abstract  We review the basic light-matter interactions and optical properties of chip-based single photon sources, that are enabled by integrating single quantum dots with planar photonic crystals. A theoretical framework is presented that allows one to connect to a wide range of quantum light propagation effects in a physically intuitive and straightforward way. We focus on the important mechanisms of enhanced spontaneous emission, and efficient photon extraction, using all-integrated photonic crystal components including waveguides, cavities, quantum dots and output couplers. The limitations, challenges, and exciting prospects of developing on-chip quantum light sources using integrated photonic crystal structures are discussed.

On-chip single photon sources using planar photonic crystals and single quantum dots

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1. Introduction

There is currently a worldwide interest in the possibility of developing single photon sources, whose unique attributes have important applications in quantum information science and give rise to novel regimes of light-matter interaction. In particular, single photon sources could lead to practical ways of doing quantum computing [1], unconditional quantum cryptography [2], quantum communications systems [3], and “completely secure” data protection. Ideally, the quantum single-photon light source should emit strictly one photon at a time, where each photon is indistinguishable (cf. a train of photons), in a high repetition rate, thus functioning as a so called “photon gun.” Although the single photon gun is a simple concept, producing trains of single photons “on demand” – or otherwise – is far from a trivial task, and many scientists have made strenuous attempts to achieve such a goal.

There are several established schemes that can facilitate the emission of single photons [4]. The easiest and most straightforward way is to attenuate pulsed lasers [5], though in this case the production of single photons is probabilistic. Therefore, there may be no photons, several photons, or many photons, since the photon number generated is subject to Poissonian statistics; this can be problematic for a number of applications such as quantum cryptography because of possible photon number-splitting attacks (“eavesdropping”) [6, 7]. Another more practical approach for obtaining single photons is to exploit spontaneous parametric down conversion in nonlinear crystals [8], where higher-frequency pump photons incident on a nonlinear crystal are occasionally split into a pair of lower-frequency...
photons; one of these photons (the “heralding” photon) is used to herald the arrival of the second photon (“heralded” photon), and thus the second photon can be well isolated and manipulated. However, there is a contradiction problem here: a larger conversion rate is usually accompanied with a greater probability of emitting two photons, while a lower conversion rate will lead to the random production of single photons (although the probability of two photon emission can be effectively reduced).

Besides single photon emission from the above macroscopic material schemes, recently there has been extensive research devoted to microscopic structures, where an isolated quantum system functions as a source for the emitted photons. In theory, a single quantum object behaving as a two-level system is an ideal single photon emitter, and there are many possible candidates that may show such behaviour, including atoms [9, 10], ions [11], molecules [12, 13], and color centres [14]. In contrast to the low attenuated lasers and heralded schemes, the single photon emitter based on a single isolated quantum system can be deterministic and on-demand, since a single two-level system can emit precisely one photon each time it is excited. Yet, many of these isolated quantum systems are impractical for a number of reasons, and can suffer from photo-bleaching and blinking, or have problems that stem from their broad optical spectrum, since the real systems are much more complicated than an idealized two-level system.

From a practical perspective, semiconductor quantum dots (QDs) [15, 16] offer an attractive material system for emitting single photons. In part due to new design insights, and due to continued improvements in semiconductor fabrication technologies, many pioneering experiments using single QD-emitted photons are now coming to the fore. Some of advantages of the QDs include: large exciton dipole moments; integrable with compact semiconductor systems [17]; fixed in position and stable; compatible with electronics and lasers. Moreover, their excited emission spectra can be nano-engineered to cover ultraviolet, visible, and infrared frequencies, rendering them fully compatible with telecom sources and components. Most single photon sources based on QDs also have excellent emission efficiencies [18]. Self-assembled QDs are nm-sized islands (typically about 20 nm in diameter and 3–5 nm in height), that are formed by having a smaller-bandgap semiconductor material embedded in a larger bandgap semiconductor material; for example, InAs embedded in GaAs. This leads to three-dimensional quantum confinement, and the electron and hole energy levels become discrete, similar to an atom. Consequently, QDs are commonly referred to an “artificial atoms”; one must keep in mind that QDs usually contain many thousands of atoms, but nevertheless they have energy levels that mimic the behavior of a two-level system.

Early optical experiments on semiconductor QDs demonstrated the well known two-level behaviour of Rabi flopping [21–23], where an excitation is driven from the ground state to the excited state and back again. However, the observed “flopping” behaviour is heavily damped, since semiconductor QDs suffer from the usual solid-state problems of environment-induced decoherence [24–27]. Decoherence has a major influence in the indistinguishable nature of the emitted photons, and is one of the biggest obstacles to overcome in realizing quantum information processing. Thus, most experiments performed to date work at cryogenic temperatures (~ 4 K is typical), yet this alone is not enough to overcome the problems of decoherence. So one must somehow have the photon emitted fast enough (before the onset of decoherence becomes significant), which can be achieved by engineering the surrounding photon density of states that the QD sees. In this regard, it is now known that spontaneous emission of an atom or QD (photon emitter) can be strongly modified by the external dielectric environment due to the Purcell effect [28], which is an example of cavity-quantum electrodynamics (cavity-QED). This is precisely why the microcavities and inhomogeneous material systems are so important. In addition, to function as a practical photon source, efficient output collection of the photons (photon extraction) is necessary, and this is one of the major advantages of chip-based quantum emitters.

The first pioneering experiments to demonstrate QD-based single photons, were carried out with single InAs QDs embedded in semiconductor micropillars [29–32] and microdisks [18, 33], which are schematically shown in Fig. 1. These structures are grown by molecular beam epitaxy on a semi-insulating GaAs substrate, followed by etching around the target QD of interest. For the micropost example, the microcavity is sandwiched between two distributed Bragg reflectors (DBRs), where the reflectivity of the bottom DBR is designed to be significantly higher than that of the top DBR, so most of the light in the cavity escapes upwards. The single QD is contained within the cavity by selective etching, and the efficiency of the emitting single photons into an output propagation mode is found to be approximately 38%. For the microdisk, the enhancement of the spontaneous emission is achieved by coupling the QD to the high-quality whispering gallery modes (WGMs);
although the QDs are distributed throughout the full disk, only those close to the excited mode’s antinode field positions are effectively coupled to the target WGM, and thus the system could generate quasi single photon pulses at a repetition rate of around 1 GHz. In both cases, the Purcell factors are in the order of 10, which means that they emit photons ten times faster than the corresponding bulk material. Experimentally, the QDs are excited by a pulsed laser, and the quantum quality of the single photons – indistinguishability – is confirmed by measuring the antibunching behaviour of the second-order quantum correlation function, which is the standard experimental proof that no two photons can be emitted at the same time.

In this review, we focus our attention on several of the main design criteria that need to be addressed when developing single photon emitters from semiconductor QDs, and show the possible solution that planar photonic crystals (PCs) offer. The term photonic crystal was introduced by Yablonovitch [19] and John [20] in their seminal works in 1987. Photonic crystals are periodic composites that allow one to engineer how photons propagate, and semiconductor-based planar PCs are of particular interest due to their ease of fabrication using standard etching and lithography techniques. Some of the main questions that we attempt to address, include: i) How to realize a sufficiently large Purcell factor so that the effects of decoherence are minimized? ii) How to enable efficient photon extraction on-chip? iii) How to have large single-photon β factors over a large bandwidth? iv) How to add in single photon functionality, such as the one photon switch? Throughout this review we discuss the underlying physics and skip the details of fabrication and experimental measurements, since these are described in many excellent experimental papers that are now appearing in the literature. In Sect. 2, we discuss the important measurement techniques commonly employed to probe single photon behaviour and efficiency. In Sect. 3, we introduce the general theory for describing the light-matter interactions and derive the exact electric field operator and the Purcell factor for arbitrary inhomogeneous dielectrics. In Sect. 4, we analyze the planar PC single cavity structure. In Sect. 5, we introduce the concept of open system cavity-QED, which is exemplified by studying the case of QD photon emission in a PC waveguide; both long waveguides and finite-size waveguides are analyzed; a connection to recent measurements is also established. In Sect. 6, we introduce practical on-chip single photon sources, by integrating couplers, cavities, and waveguides. In Sect. 7, we discuss the integrated side-coupled-cavity waveguide system, and introduce the phenomena of single photon switching and electromagnetic induced transparency. In Sect. 8, we give our conclusions and a brief outlook into the future.

2. Quantum statistics and measurement aspects of single photons

According to the photon time distribution, photon sources can be classified into three types: a) bunching, b) random, and c) antibunching; these distributions correspond to three varieties of photon number distribution, namely Bose-Einstein, Poisson, and sub-Poisson or single photon (see Fig. 2). These statistical properties can be accurately characterized by the second-order correlation function (or intensity correlation function) \( g^{(2)}(\tau) \) – the conditional probability of detecting a photon at \( t + \tau \), conditioned on detecting one at \( t \). After normalizing by the average probability of emitting a photon at any time, one obtains the normalized second-order correlation function for stationary fields:

\[
\frac{g^{(2)}(\tau)}{g^{(2)}(0)} = \frac{\langle I(t)I(t+\tau) \rangle}{\langle I(t) \rangle^2},
\]

Depending upon the source class, such as thermal (Bose-Einstein distribution) or coherent (Poisson distribution), the numerator in the above equation is always larger than the denominator: \( \langle I^2 \rangle \geq \langle I \rangle^2 \), because any classical stationary random function of time must satisfy the Schwartz inequality, and so \( g^{(2)}(\tau) \geq 1 \). To describe the quantum property of a single photon source, one must use the second-order quantum correlation function [34, 35], defined as

\[
g^{(2)}(t, \tau) = \frac{\langle I(t)I(t+\tau) \rangle}{\langle I(t) \rangle^2} = \frac{\langle \hat{a}^\dagger(t)\hat{a}^\dagger(t+\tau)\hat{a}(t+\tau)\hat{a}(t) \rangle}{\langle \hat{a}^\dagger(t)\hat{a}(t) \rangle^2},
\]

where \( \hat{a}^\dagger \) (\( \hat{a} \)) is the creation (annihilation) operators of the mode being measured and ‘:’ denotes the normal ordering of the operators. For a sub-Poissonian source, such as the attenuated laser, \( g^{(2)}(\tau = 0) < 1 \), which has a reduced multiphoton probability as compared to coherent light with Poissonian photon statistics. For a true single photon source, the value of \( g^{(2)}(\tau = 0) \to 0 \), since two effective detection events for a single photon state \( |1 \rangle \) yields 0. Generally, the closer to \( g^{(2)}(\tau = 0) \to 0 \), the better the single photon source, and this is a common experimental target.
3. Exact quantum field operator and the Purcell effect for a QD in an arbitrary inhomogeneous dielectric

As we have alluded in the introduction, there are three essential criteria for producing practical single-photon sources: i) sufficient suppression of the effects of decoherence or pure dephasing during the photon emission process; ii) efficient excitation of the photon emitter (QD); and iii) efficient extraction and thus manipulation of the emitted photons. The first criterion can be achieved by placing the QDs in an environment that increases the local photon density of states (LDOS) through the Purcell effect. The Purcell factor is a measure of the vacuum-induced spontaneous emission rate enhancement, defined by $I' / I_0$, where $I'$ (the Einstein A coefficient) is the spontaneous emission rate associated with population decay rate from an excited state to the ground state, and $I_0$ is the spontaneous emission rate in a homogeneous medium with some nominal background refractive index.

The classical electrodynamical properties can be obtained from Maxwell’s equations. To describe the general theory of light emission, we first define the photon Green function and the eigenmodes of the system. The two-space point function $G(r, r'; \omega)$ is the electric-field Green function that describes the field response at $r$ to an oscillating polarization dipole (source) at $r'$, as a function of frequency; the Green function is classical and is defined from

$$\left[ \nabla \times \nabla \times -\frac{\omega^2}{c^2} \varepsilon(r) \right] G(r, r', \omega) = \frac{\omega^2}{c^2} \delta(r - r'),$$

where $\mathbf{1}$ is the unit dyadic. The Green function $G = G^T + G^L$, contains contributions from both transverse and longitudinal modes, and can be written as $G(r, r'; \omega) = \sum_\lambda \left( \omega^2 f_\lambda^T(r) f_\lambda^T(r') / (\omega^2_\lambda - \omega^2) - f_\lambda^L(r) f_\lambda^L(r')^* \right)$, where $f_\lambda^T$ and $f_\lambda^L$ are the normalized transverse and longitudinal modes of the medium corresponding to $\nabla \cdot D = 0$ and $\nabla \cdot D \neq 0$, respectively, where $\mathbf{D}$ is electric displacement field. The transverse modes, $f_\lambda^T \equiv f_\lambda$, are solutions of non-zero eigenvalue: \[ \nabla \times \nabla \times -\omega^2_\lambda / \varepsilon(r) \mathbf{f}_\lambda(r) = 0. \] These modes are easily computed for most cavities and PC media using standard electromagnetic techniques, and one can also frequently compute the numerically-exact Green function. Another useful Green function is defined through $K(r, r'; \omega) = \sum_\lambda \omega^2_\lambda f_\lambda(r) f_\lambda(r')^* / (\omega^2_\lambda - \omega^2)$, which, like $G^T$, only depends on the transverse modes of the system. Note that we have exploited the relation, $\mathbf{1} \delta(r - r') = \sum_\lambda [f_\lambda(r) f_\lambda^*(r') + f_\lambda^T(r) f_\lambda^T(r')] \varepsilon(r + r')$. The following...
relationships can also be derived: \( \mathbf{K} = \mathbf{G} + \mathbf{1} \delta (\mathbf{r} - \mathbf{r}') / \varepsilon (\mathbf{r}) \), and \( \mathbf{K} = \mathbf{G}^T + \delta^T (\mathbf{r}, \mathbf{r}') / \varepsilon (\mathbf{r}) \), where \( \delta^T (\mathbf{r}, \mathbf{r}') = \sum_\lambda f_\lambda (\mathbf{r}) f^*_\lambda (\mathbf{r}') \varepsilon (\mathbf{r}) \) is a generalized transverse delta function [38].

Next, we introduce some of the basic quantum mechanics. To describe quantized light emission in an arbitrary inhomogeneous dielectric, we employ a canonical Hamiltonian of the form [38, 39] (the time dependence is implicit):

\[
\hat{H} = \hbar \omega_z \hat{\sigma}_z^x / \sigma_x^+ \sigma_x^+ + \sum_\lambda \hbar \omega_\lambda \hat{a}_\lambda^\dagger \hat{a}_\lambda \\
- i \hbar \sum_\lambda \left( \hat{\sigma}_z^x + \hat{\sigma}_z^+ \right) \left( \hat{g}_\lambda \hat{a}_\lambda - \hat{g}_\lambda^* \hat{a}_\lambda^\dagger \right),
\]

where \( \hat{a}_\lambda \) represents the field mode operators, \( \sigma_x^\pm \) are the Pauli operators of the QD excitons (where we assume one QD exciton in the frequency regime of interest), and the QD-photon interaction terms are treated within the dipole approximation (valid for photon emitters whose spatial size is much smaller than a wavelength); in addition, \( \omega_z \) is the resonant frequency of the QD, \( \omega_\lambda \) is the eigenfrequency corresponding to the transverse modes of the system (\( f_\lambda (\mathbf{r}) \)) excluding the dots; and \( g_\lambda \) is the field-dot coupling coefficient: \( g_\lambda = \sqrt{\frac{e^2}{2\pi}\mu} f_\lambda (\mathbf{r}_d) \), where \( \mu = \mathbf{n}_{\mu} \) is the dipole moment of the QD electron-hole pair, and \( \mathbf{n}_{\mu} \) is the dipole unit vector; the QD is at the spatial position \( \mathbf{r}_d \). The Heisenberg equations of motion for the operators can be derived from \( \hat{O}_\lambda = -i \hbar^{-1} [\hat{O}_\lambda, \hat{H}] \), yielding

\[
\frac{d \hat{a}_\lambda}{dt} = -i \hbar \omega_\lambda \hat{a}_\lambda + \hat{g}_\lambda \left( \hat{\sigma}_z^x + \hat{\sigma}_z^+ \right),
\]

\[
\frac{d \hat{a}_\lambda^\dagger}{dt} = i \hbar \omega_\lambda \hat{a}_\lambda^\dagger + \hat{g}_\lambda \left( \hat{\sigma}_z^x + \hat{\sigma}_z^- \right),
\]

\[
\frac{d \hat{\sigma}_z^x}{dt} = -i \hbar \omega_\lambda \hat{\sigma}_z^x - \sum_\lambda \left( \hat{g}_\lambda \hat{a}_\lambda - \hat{g}_\lambda^* \hat{a}_\lambda^\dagger \right),
\]

\[
\frac{d \hat{\sigma}_z^+}{dt} = i \hbar \omega_\lambda \hat{\sigma}_z^+ + \sum_\lambda \left( \hat{g}_\lambda \hat{a}_\lambda - \hat{g}_\lambda^* \hat{a}_\lambda^\dagger \right).
\]

The interpretation of the above quantum dipole is similar to what occurs classically, where the bare polarizability is medium-dressed or renormalized by the surrounding medium, which then properly accounts for radiative coupling to the photon modes. The subtle difference between the quantum and classical fields, is that in the quantum case, the dipole source above stems from quantum noise or vacuum fluctuations, and the Green function \( \mathbf{K} \) appears, rather than \( \mathbf{G} \) [38]. We can rearrange the quantum field operator to better connect to the more familiar self-energy term, \( \Sigma (\omega) \):

\[
\hat{\mathbf{E}} (\mathbf{r}, \omega) = \frac{i \mathbf{K}(\mathbf{r}, \mathbf{r}_d; \omega) \cdot \mathbf{\mu}}{\varepsilon_0} \\
\times \frac{\left[ \hat{\sigma}_z^x (t = 0)(\omega + \omega_x) + \hat{\sigma}_z^+ (t = 0)(\omega - \omega_x) \right]}{\omega^2 - \omega - \omega \Sigma (\omega)},
\]

where \( \omega \Sigma (\omega) = \frac{2 \mu^2 \omega_x \mathbf{n}_d \cdot \mathbf{K}(\mathbf{r}_d, \mathbf{r}_d; \omega) \cdot \mathbf{n}_d}{\hbar \varepsilon_0} \). The imaginary part of the self-energy defines the spontaneous emission rate

\[
\Gamma (\mathbf{r}_d, \omega) = \frac{2 \mu^2 \mathbf{n}_d \cdot \text{Im} \left[ \mathbf{K}(\mathbf{r}_d, \mathbf{r}_d; \omega) \right] \cdot \mathbf{n}_d}{\hbar \varepsilon_0} \\
= \frac{2 \mu^2 \mathbf{n}_d \cdot \text{Im} \left[ \mathbf{G}(\mathbf{r}_d, \mathbf{r}_d; \omega) \right] \cdot \mathbf{n}_d}{\hbar \varepsilon_0},
\]

which would be the decay rate of the population from an upper excited state to ground, with an initial quantized field in vacuum; therefore, the process of spontaneous emission can be thought of as stimulated emission from the vacuum, and thus both spontaneous and stimulated emission have the same quantum mechanical origin. For obtaining the spontaneous emission rate, evidently one can work with \( \mathbf{K}, \mathbf{G} \), or \( \mathbf{G}^T \) above. Note also that the terminology of spontaneous emission “rate” only makes sense for a weak or intermediate coupling regime. We stress, however, that the general formalism above rigorously covers both weak and strong coupling regimes in a self-consistent way, naturally recovering phenomena such as vacuum Rabi oscillations and reverse spontaneous emission [40–42]. Our expression for the spontaneous emission is also in agreement with derivations from Dung et al. [43, 44] and Wubs et al. [38], who have pioneered many of the general theories of quantized light emission within inhomogeneous materials. In a similar manner, one can also obtain a frequency shift of the QD excitation, defined through \( \Delta \omega = \mu^2 \mathbf{n}_d \cdot \text{Re} \left[ \mathbf{K}(\mathbf{r}_d, \mathbf{r}_d; \omega) \right] \cdot \mathbf{n}_d / \hbar \varepsilon_0 \). This contribution includes an electronic vacuum Lamb shift [45], as well as the inhomogeneous photonic contribution, which would typically be the dominant and measurable contribution. Since computation of the vacuum Lamb shift requires a careful mass renormalization formalism, it is more convenient and practical to compute the photonic contribution

\[
\text{Im} \left[ \mathbf{G}(\mathbf{r}_d, \mathbf{r}_d; \omega) \right] \cdot \mathbf{n}_d / \hbar \varepsilon_0.
\]
from the inhomogeneous structure, and subtract off the homogeneous contribution.

We next introduce some convenient approximations for a planar PC medium, in which a well defined transverse mode is the dominant contribution. Since $G^T(r, r'; \omega) = K(r, r'; \omega) - \delta^T(r, r')/\varepsilon(r)$, then $G^T_m \approx K_m$, where $m$ represents the dominant modal contribution to the Green function; e.g., $m = c$ or $m = w$ could be the cavity or waveguide mode Green function that dominates as it appears within a photonic bandgap. This allows us to write out the \textit{dominant modal contribution} to the decay rate and frequency shift, respectively, as

$$\Gamma(r_d, \omega) \approx \frac{2\mu^2}{\hbar} n_d \cdot \text{Im}[\text{Re}[G^T_m(r_d, r_d; \omega)]] \cdot n_d$$

and

$$\Delta \omega(r_d, \omega) \approx \frac{\mu^2}{\hbar} n_d \cdot \text{Re}[\text{Im}[G^T_m(r_d, r_d; \omega)]] \cdot n_d.$$ (14)

Finally, we obtain a formal expression for the Purcell effect factor:

$$F(r_d, \omega) = \frac{n_d \cdot \text{Im}[G^T_m(r_d, r_d; \omega)] \cdot n_d}{n_d \cdot \text{Im}[G^T_{\text{nom}}(\omega)] \cdot n_d},$$ (15)

where $n_d \cdot \text{Im}[G^T_{\text{nom}}(\omega)] \cdot n_d = \omega^3 \sqrt{\varepsilon_b} / (6\pi\varepsilon_b^3)$, with $\varepsilon_b$ the dielectric constant of a homogeneous material.

4. Single photon emission within a PC cavity

Advanced lithography and etching techniques have recently led to major improvements in the fabrication of nanoscale PCs; most notably has been the spectacular increase in the quality factor ($Q$) and decrease in effective mode volume ($V_{\text{eff}}$) of planar PC nanocavities [46]. These nanoscale cavities are enabling the study of fundamentally new regimes of light-matter interaction, such as single-exciton QED [47]. Therefore, the most common PC structure to date for trying to demonstrate a single photon source is the PC cavity [48-50].

A simple PC cavity can be realized by adding a defect into the perfect PC slab, for example, by removing one hole. Moreover, it has been demonstrated, through new design insights, that very large $Q/V_{\text{eff}}$ values can be realized. In Fig. 5, we show the popular L3 cavity [46], based on the design of Akahane et al. [46]. The physics of this cavity mode is well established: light is trapped by the high-index-contrast PC in the $x$- and $y$-directions, and bound as much as possible by total internal reflection in the $z$-direction. Coupling of the field mode to radiation modes above the light line causes decay of the field within the cavity; this coupling is minimized by moving two of the holes [46], resulting in a large $Q$. These large $Q/V_{\text{eff}}$-cavities were first demonstrated for silicon membranes, but recent experiments on InP [51] and GaAs [52] cavities, show that the fabrication quality with these III-V materials is becoming comparable with silicon.

To maximize the Purcell effect, one needs to have the QD spatially positioned near the antinode of the localized cavity mode field. Using randomly arranged self-assembled QDs, $30\text{ nm}$ accuracy can be achieved by strategically monitoring the QD position via atomic force microscopy (AFM) technology [50]. In this way, QDs within the center of the slab membrane may be located, prior to writing the PC with e-beam lithography. Certainly, a major on-going challenge that needs to be addressed, experimentally, is how to place the QDs in the position of interest, in a more \textit{deterministic} manner, but progress is being made.

The spontaneous emission of the target QD exciton will be enhanced if it is coupled to the cavity mode, both spatially and spectrally; for an uncoupled QD, the emission will be inhibited due to the presence of the slab and the photonic bandgap. Assuming that the microcavity supports a single cavity mode, the transverse Green function of the monolithic cavity can be expressed as $G^T_c(r, r'; \omega) = \omega^2 \ell_{c}(r) \ell^*_c(r') / (\omega^2 - \omega^2 - i\omega T_0)$, where $\omega_c$ is the resonance frequency of the cavity mode, which is deep inside the photonic bandgap; in addition, $\ell_{c}(r)$ is the localized eigenmode normalized through $\int dV \varepsilon_c(r) \ell_{c}(r) \ell^*_c(r) = 1$, and $T_0 = \omega_c/Q$ is the cavity linewidth rate. Another useful cavity parameter is the effective mode volume, which can be defined from the cavity eigenmode $\ell_{c}(r)$: $V_{\text{eff}} = 1/\max[\varepsilon_c(r) \ell_{c}(r) \ell^*_c(r)]$. It should be noted that this expression is only approximately true for leaky modes. Thus, using the earlier results of the theory section, the enhancement of spontaneous emission for a QD resonant with a cavity mode can be expressed as

$$F = \frac{6\pi A Q}{V_{\text{eff}}} \left( \frac{c}{\sqrt{\varepsilon_c \omega}} \right)^3,$$ (17)

where $A$ is a coupling constant related to the deviation of the QD from the antinode (i.e., $A = 1$ if the QD is precisely placed at the field antinode).

From the above approximate formulas, and using physically intuitive parameters such as the $Q$ factor and the
local electric field at the field antinode, we can obtain some qualitative results for the Purcell factor. Further rigorous calculations of the spontaneous emission can be calculated by carrying out a direct numerical computation of the photon Green function [53], by solving the Maxwell equations, excited by a polarization dipole source. To do this, for example, one can use FDTD (finite-difference time domain) techniques [54]. In FDTD, the electric field is excited by a dipole source \( p \), which satisfies

\[
\nabla \times \frac{1}{\mu(\omega, \mathbf{r})} \nabla \times \frac{\omega^2}{c^2} \varepsilon(\omega, \mathbf{r}) \mathbf{E}(\mathbf{r}, \omega) = \frac{\omega^2}{c^2} p(\omega) \delta(\mathbf{r} - \mathbf{r}'),
\]

where \( \varepsilon(\omega, \mathbf{r}) / (\mu(\omega, \mathbf{r})) \) is the frequency-dependent permittivity (permeability). Subsequently, the \( ii \)-th component of the numerical Green function is

\[
G_{ii}(\mathbf{r}, \mathbf{r}; \omega) = \frac{\text{FFT}[E_{ii}^p(\mathbf{r}, t)]}{\text{FFT}[p_{ii}^l(\mathbf{r}, t)]} = \frac{E_{ii}^p(\mathbf{r}, \omega)}{p_{ii}^l(\mathbf{r}, \omega)},
\]

where FFT is a Fourier transform of the numerical fields obtained by FDTD. The precise accuracy is dependent on the grid size in the simulation volume; however, for sufficiently small spatial grids, the results are found to be very accurate. For example, using a grid size that is 1/25 of the central wavelength \( \lambda_0 \) of the incident pulse, we have checked against known analytical solutions that the error is less than 1% over a broadband frequency range of 0.5–1.3 \((2\pi c/\lambda_0)\).

We first investigate the case of a single photon source based on a L3 PC cavity shown in Fig. 5. The structural parameters used in this simulation are as follows: semiconductor slab index \( n = 3.464 \), lattice constant \( a = 420 \) nm (PC pitch), two-hole outward shift length of 0.15\( a \), slab thickness and hole radius are 0.5\( a \) and 0.275\( a \), respectively. The TE-like band-gap ranges from 185 to 228 THz – or 0.26–0.32\( c/a \) in scaled frequency units.

In Fig. 6, we show the Purcell factor for a perfectly positioned and aligned QD (maximum coupling), which is calculated directly by the 3D FDTD simulation. Therefore, we are actually showing the “exact” Purcell factor (at least in the absence of strong coupling). The Purcell factor for the \( y \)-polarized exciton is as high as \( F_y = 3385 \) at the resonant frequency of 192.78 THz. We emphasize that the theoretical maximum Purcell factor of 3385 is the ideal enhancement rate of spontaneous emission, because several ideal conditions are assumed; for example, the QD is maximally coupled to the antinode of the cavity mode and perfectly resonant with the cavity mode frequency. The concept of a spontaneous emission rate – as a measure of spontaneous emission – only make sense for weak to intermediate coupling regimes, which depends upon the dipole moment of the emitter as well as the material coupling parameters.

For typical self-assembled QDs, assuming maximum coupling as above, then we would already be in the strong coupling regime. Therefore, in general one can think of the Purcell factor here as representing the properties of the dielectric environment without a self-consistent coupling to the emitter, i.e. the LDOS or the projected LDOS. We stress, however, that the above Green function theory can be used to study weak to strong coupling regimes without any change in formalism [55, 56]. In reality, sample imperfections and finite material loss will also reduce this estimated maximum value, as discussed in more detail below.

For this peak enhancement frequency, the corresponding electric-field distribution of the \( y \)-polarized resonant mode, at the center of the slab, is shown in Fig. 7. The quality factor and the effective volume are 31 432 and 0.0627\( \mu m^3 \), respectively. With these parameters, the approximate Purcell factor can also be calculated from Eq. (17), which yields 3981, and is notably larger than 3385. Therefore, the usual formula for the Purcell factor (Eq. (17)) can only be used as an approximation; the origin of this discrepancy is that the confined mode is a quasi-mode or leaky-mode, where the mode can leak out over larger volumes that what would be estimated in a finite-size numerical calculation. In most realistic devices, the spatial positions of the QDs will always deviate from the electric-field maximum; for example, if located at 90% of the peak electric field, then the Purcell factor enhancement will be reduced to about 80% of its peak value; in addition, the enhancement also suffers from fabrication imperfections, that can partly reduce the quality factor [57, 58] and can also cause
a shift in the resonance of the cavity mode [59, 60]. For comparison, we also calculate the emission properties of the no-shift cavity (see Fig. 6b), which gives a much smaller Purcell factor of only 624, and a quality factor of 4856. For experiments in the intermediate-to-strong-coupling regime, with self-assembled QDs, Hennessy et al. [50] investigated the quantum nature of a QD embedded in a similar PC cavity to that shown above. For a large off-resonant case, with a detuning of 4.1 nm (5.7 meV), the exciton lifetime $\tau_x$ is increased to 8.7 ns from 1 ns (the exciton lifetime in a bulk material). While for the resonant case ($\omega_c \approx \omega_x$), $\tau_x$ decreases rapidly to only 60 ps, which is only about $\frac{1}{14}$ of the off-resonant value. This certainly confirms that pronounced Purcell effects can be realized. A related theory of the spectral observations and second-order quantum correlation measurements are presented elsewhere [56, 61] (see also [62–64]), which in general show the need for a medium-dependent theory, namely a theory that is based on quantizing the medium-dependent macroscopic electric field operators; using such a quantization procedure, the relationship between the electric field operators and the exciton operators is established directly with the medium-based Green functions; this helps to clarify the underlying physics of photon emission processes in these chip-based structures, since unexpected observations can occur such as off-resonance excitation of the cavity mode [50, 56].

5. Single photon source using PC waveguides

5.1. The infinite-size PC waveguide: open system cavity-QED

One major drawback of using the monolithic quasi closed-cavity is that the photons vertically emitted out of the cavity cannot be efficiently collected and manipulated. Moreover, it is against the general vision of planar integration, and one ultimately desires to emit the photon on-chip, into a target propagating mode. This is exactly where the unique advantages of planar PCs come in; compared to other microcavity systems such as the micropillars and microdisks, PC waveguides have the inherent advantage that they can collect and control the photons on-chip [65–68]. In fact, enhanced spontaneous emission does not even need a quasi-closed cavity such as that shown with the L3 cavity, and open system cavity-QED can be exploited to achieve an LDOS enhancement by appropriate bandgap engineering of the propagation modes.

Kleppner predicted in 1981 [69], that photon emission rates can be enhanced using cylindrical metallic-wire waveguides, for emission frequencies lying near the mode bandedge. Although these modes are usually lossy, the idea introduced the general concept that single photon emission into a photonic wire is both rich in physics and applications. The photonic wire, such as a bar of silicon surrounded by air, uses the high-index-contrast to confine light at selected wavelengths, using the principle of total internal reflection. By embedding a photon emitter at the center of such a photonic wire [70], significant collection efficiencies can be achieved: $\sim 50$–$80\%$ into a bound propagation mode, however, there is very little spontaneous emission enhancement which is necessary for solid-state single photon emission applications to overcome the known problems of decoherence. The relationship between slow-light modes and spontaneous enhancement can be obtained from the Green function of the PC waveguide structure [71], and is discussed in some detail below.

We first assume an ideal structure (no disorder or finite-size effects), described through the spatially-dependent dielectric constant $\varepsilon(r)$, as schematically illustrated in Fig. 8a; and although this only shows eight unit cells along the waveguide axis, keep in mind that we modeling an infinite structure (in the $x$-direction). The structure supports a Bloch waveguide mode (cf. the solid curve in the band structure plot), which, in the region below the light line, is lossless in the absence of disorder: the ideal group velocity is then $v_g = d\omega/dk$. The physics of PC waveguides is rather intriguing, with the defining feature that their engineered waveguide band dispersions yield a vanishing group velocity [72–74].

In Fig. 8b we show an example of the planar PC waveguide band structure, which displays the two lowest order propagation modes. For PC propagation modes below the
light line, described by \( f_k(r) = \sqrt{T_k} e^{ikz} \), the waveguide Green function can be expressed as

\[
G^T_w(r, r' ; \omega) = \sum_k \frac{\omega^2}{\omega^2 - \omega^2} f_k(r) f_k^*(r').
\] (20)

Replacing the \( k \)-summation (\( k \equiv k_g \)) by an integral, i.e.,

\[
\sum_k \rightarrow \frac{1}{\pi} \int dk,
\]

where \( L \) is the length of the waveguide,

\[
G^T_w(r, r' ; \omega) = \frac{L}{4\pi} \int_{-\infty}^{\infty} dk \frac{\omega}{\omega - \omega - i\delta} f_k(r) f_k^*(r'),
\] (21)

with \( \delta \) a positive infinitesimal variable. After carrying out the complex pole integration,

\[
G^T_w(r, r' ; \omega) = \frac{i\omega}{2v_g} \left[ \Theta(x - x') e_{k_w}(r) e_{k_w}^*(r') e^{i\omega(x - x')} + \Theta(x' - x) e_{k_w}(r) e_{k_w}^*(r') e^{-i\omega(x - x')} \right],
\] (22)

where we treat \( v_g \) as positive, and \( \Theta(x - x') \) is the Heaviside function. Note also that \( K_w = G^T_w \) above, because of the continuum nature of the waveguide modes \( (\omega_k = \omega) \); so one can work with either function. Since the Purcell factor (s. Eqn(G)) is inversely proportional to the group velocity \( v_g \), slow light modes will exhibit pronounced enhanced emission [71]; and with typically significantly more bandwidth than that might be achieved by using nanowand defect cavity modes.

Interestingly, the Purcell factor tends towards infinity at the waveguide band edge. However, this divergence never happens in real structures primarily because of fabrication imperfections; this is not necessarily a disaster, since there are certainly ways to make useful devices once one understands and accounts for imperfections as well. For example, Hughes et al. [75] introduced a PC wave-guide model for incoherent extrinsic scattering in the slow-light regime. With \( v_g \) the group velocity, they predicted that the ensemble average backscattering loss for weak perturbative disorder, scales as \( 1/v_g^2 \) and dominates over scattering into radiation loss modes (1/vg scaling). Similar scalings have been found by Povinelli et al. [76] and by Gerace and Andreani [77]. These general approximate loss-scaling relations have now been confirmed experimentally by a number of groups, e.g. [78–80], except that they naturally break down at extremely low group velocities where using the ideal band structure is no longer a good approximation [81].

In this case, the effective group velocity will be noticeably altered from the ideal value for around \( v_g \lesssim c/35 \), and will have a minimum of around \( c/50 \) \( \approx c/200 \); related effects also occur in coupled-cavity structures using PC cavities [59], and we refer the interested reader to the works of, for example, Fussell et al. [59], Engelen et al. [80], and Patterson et al. [82]. For the present purpose, we note that fabrication disorder introduces a minimum achievable group velocity and thus a maximum Purcell effect, and the nominal lossless waveguide modes become very lossy in the regime of slow light (where multiple scattering effects begin to dominate [82]). This suggests that long waveguides, operating in the slow light regime, will be impractical for single photon applications; we will come back to this point later.

A key design parameter for efficient photon extraction of a single photon is the single photon \( \beta \) factor, which is related to the collection efficiency of the output photon into a desired mode. The \( \beta \) factor was traditionally proposed in order to quantify the lasing efficiency from spontaneous emission into a desired lasing mode. Here, we define the single-photon \( \beta \) factor as the fraction of the emitted light that goes into a target waveguide mode, i.e. the probability of a photon being emitted into a waveguide mode. Neglecting the possible impact of extrinsic scattering losses (but keeping in mind the minimum group velocity), the \( \beta \) factor for a planar PC waveguide can be written as [83]:

\[
\beta = \frac{\Gamma_{\text{bound}}}{\Gamma_{\text{bound}} + \Gamma_{\text{rad}} + \Gamma_{\text{nr}}},
\] (23)

where \( \Gamma_{\text{bound}} \) is the decay rate into the target bound waveguide mode, \( \Gamma_{\text{rad}} \) is related to the decay of the spontaneous emission into a radiation modes (modes above the light line), and \( \Gamma_{\text{nr}} \) is the nonradiative decay rate due to various dephasing processes. For low temperatures of around 4 K, the non-radiative relaxations typically occur on a much longer time scale than radiative transitions (especially in an enhanced emission regime). Therefore, the \( \beta \) factor can be simplified to \( \beta = \Gamma_{\text{bound}}/(\Gamma_{\text{bound}} + \Gamma_{\text{rad}}) \), which is valid for low temperature operation.

Recently we have presented an analytical Green-function formalism to calculate the Purcell factor and the \( \beta \) factor of an ideal planar PC waveguide [83], which extends earlier work on the Purcell factor alone [71]. Using the structure depicted in Fig. 8, we show in Fig. 9a an example of the electric-field distribution of guided mode \( f_k \)-component with a small group velocity (at \( k_w = 0.47 \text{ (2/}\mu \text{m)} \)), with \( v_g \sim c/154 \) for one unit-cell of the waveguide, where most of the energy is localized within the center of the slab. This wave vector was chosen as a typical upper \( k \) limit (smaller group velocity) that can be measured experimentally [84, 85]. The corresponding effective mode volume (per unit cell [83]) is only \( V_{\text{eff}} = 0.03 \mu \text{m}^3 \). The mode-volumes of the broadband waveguide modes range from 0.02–0.03 \( \mu \text{m}^3 \), which correspond to \( k \) values of 0.3–0.48 (2/\mu m). An example spatial map of the Purcell factor for a \( y \)-polarized dipole (see Fig. 9b) is also presented as a function of QD position, on a plane parallel to the slab for \( k_w = 0.47 \text{ (2/}\mu \text{m)} \). Substantial enhancements in the spontaneous emission rate by up to a factor of 30 are achievable, which will become even greater for smaller group velocities. As can be seen, if the same dipole was placed away from the anti-node position, the Purcell factor would reduce substantially as it scales with the field strength squared. Hence there is a “hot spot” area of around \( \pi(80 \text{ nm})^2 \) that yield
Purcell factors of greater than 20. The peak Purcell factors and $\beta$ factors as a function of frequency are shown in Fig. 9c,d. Most remarkably, is the fact that enhanced $\beta$ factors ($>0.9$) can be maintained with a substantial bandwidth ($>10$ THz or 60 nm), which is in stark contrast to the usual cavity-QED regimes that exploit narrowband cavities. Similar findings were reported by Lecamp et al. [86], who also exploited periodic Bloch mode analysis. Related studies by Fussell et al. investigated the strong coupling regime between a QD and a lossy coupled-cavity waveguide by the tight-binding approach [87]; however, later investigation in the present of fabrication disorder [59], concluded that strong coupling effects would likely be destroyed because of a significant broadening of the LDOS at the waveguide bandedge.

With regards to these theoretical predictions, some related experiments have been performed. In 2005, Viasnoff-Schwoob et al. [88] presented evidence for the spontaneous emission enhancement of QD exciton decay coupled to a leaky mode in a planar PC waveguide, using out-of-plane and in-plane light emission measurements. More recent experiments on bound waveguide modes below the light line have been carried out by Lund-Hansen et al. [89], who observe enhanced emission in the slow-light regime, and find good agreement with our analytical formulas presented above. Specifically, Lund-Hansen et al. performed time-resolved spontaneous emission measurements on single QD positioned within the planar PC waveguide, where the QDs are excited by a pulsed Ti:Sapphire laser. Purcell factors of up to 27, and $\beta$ factors of up to 0.89 were demonstrated; moreover, an impressive $\beta$ factor greater than 0.5 was observed over a bandwidth of 20 nm.

In the next sections, we will highlight a few chip-based schemes that can facilitate a much richer control of cavity-QED-enabled light sources. Before doing that, we make a few remarks about possible experimental bottlenecks and highlight the need for experimentalists and theorists to work together. First of all, we note that each PC sample is different, even without any QDs; so an ensemble average of measurements on nominally identical samples is needed to allow one to understand what kind of structures are being made and with what precision; following an understanding of the general dispersive, loss, and input-output characteristics, one can feed this information back into the general theoretical understanding of the device, even before QDs are embedded. Important aspects to understand include the broadened LDOS enhancements, the minimum group velocity, the finite-length dependence, and the essential role of fabrication imperfections, all of which can be added into the medium-dependent Green function. Second, comes the strategic coupling to QDs. A general ensemble of QDs can largely spoil the enhanced light-matter coupling of any PC system, and this applies to waveguides and cavities; for example, spectrally uncoupled (to the cavity mode) excitons can still radiatively couple to the environment and, for example, reduce the quality factor of the cavity. Thus the spatial positioning of a target QD, and a background ensemble of QDs can play a qualitative role. Spectrally tuning the PC system and target QD exciton is also important, and this can be achieved, for example, by gas condensation or temperature tuning; however, the latter brings about an increased influence of non-radiative decay and electron-phonon coupling. These are only a few of the important attributes; other include: input coupling, target temperature operation, entangled photon pair generation, coherent pulse generation, target wavelength operation, and so on, so we anticipate much room for improvement. Taken together, it seems that a systemic investigation is required, which will involve design and redesign, iterating towards closely matching theory and experiments in a deterministic way. Below we highlight some other approaches beyond the single cavity and single waveguide approach, that may help in this regard.

5.2. The “finite-size” PC waveguide

We next address the question of what happens when only a short section of a finite-size waveguide is considered, where the rigorous Bloch mode analysis and the analytical Green function techniques do not apply. In Figs. 10a and b, we show the numerically calculated spontaneous emission factors as a function of frequency for a ten unit-cell planar PC waveguide; the waveguide is bounded by, respectively, perfectly matched layers (PML) and an air medium, on both ends of the waveguide for two different
Figure 10  (online color at: www.lpr-journal.org) (a) Purcell factors ($F$) versus frequency for a QD embedded in a 10 unit-cell finite size waveguide surrounded by PML. Dashed (solid) line is for the case when the dipole orientation of the QD is along the $x$-($y$) axis, and the dotted vertical line shows the bandedge of the fundamental waveguide mode of an infinite structure. (b) Same as in (a) but with the waveguide ends surrounded by air. (c) Center of the slab contour of the $|f_y|^2$ field distribution corresponding to the peak Purcell factor shown in (b), $x$-axis is along the waveguide and point to the right, $y$-axis is in plane and perpendicular to $x$-axis. (d) As in (c) but for the $|f_x|^2$ component of the electric field.

dipole orientations. The PML allows light to propagate out of the sample without any reflections. Remarkably, even with only ten unit cells, a comparison of the spontaneous emission factors for both $y$- and $x$-oriented dipoles reveal all the main features of the infinite-size band structure, such as LDOS enhancements whenever the group velocity approaches zero (flat bands). With finite size, however, the band-edge divergences are significantly broadened, and also the LDOS peaks are blue shifted from the ideal band structure (shown in the figure with a vertical dotted line).

When one includes air as a surrounding medium on both sides of the PC waveguide, there is a substantial enhancement of the emission-factor ($F$), and additional spectral peaks arise due to Fabry-Perot resonances. For the LDOS peak indicated in Fig. 10b, the corresponding $y$-polarized mode ($x$-polarized mode) at the center of the slab is shown as a contour image in Fig. 10c [Fig. 10d], which yields a calculated effective mode volumes of $V_{\text{eff}} \approx 0.1 \mu\text{m}^3$. The spatial dependence of the spontaneous emission essentially follows the spatial profile of the field modes. This suggests that QD spatial positioning within 20–40 nm will be key in achieving the predicted Purcell factor enhancements. We also highlight that these large enhancements can be achieved for both $x$- and $y$-polarization components, although the peak frequencies will be different [see solid and dashed curves in Fig. 10]. We add the following two remarks: a) the influence of the facet terminations can also play an important role [90], which could allow a systematic investigation of QD coupling as a function of sample length; and b) similar features in the PC waveguide LDOS have been observed experimentally [91].

6. Practical on-chip single photon sources

6.1. Single photon source using a PC waveguide and an output coupler

Stimulated by the enhanced Purcell factors of a PC waveguide and efficient on-chip unidirectional collection of the emitted photons, our group has also recently proposed a “single photon gun” based on a small section of a PC waveguide near the slow-light mode, integrated with an output coupler. The coupler is based upon the design of Banaee et al. [92], and the schematic structure is shown in Fig. 11. As an example, the Purcell factor versus frequency for a 10 unit-cell waveguide in a unidirectional output is shown in Fig. 12, where the peak value is about 46. With realistic parameters, a 20 unit-cell waveguide can achieve Purcell factors of more than 200 [90]. The electric-field distribution at the peak Purcell factor (see Fig. 12) is shown in Fig. 13.

Figure 11 (online color at: www.lpr-journal.org) Schematic diagram of a single photon source based on a finite-size PC waveguide, in the slow light regime; the green dot refers to the QD, and the target photon is emitted on-chip via the waveguide mode.

Figure 12 (online color at: www.lpr-journal.org) Purcell factor ($F_y$) versus frequency for a QD embedded in the finite-size PC waveguide (cf. Fig. 11).
With the integrated coupler, more than 60% of the emitted field overlaps with the output waveguide mode ($\beta > 0.6$). To obtain these parameters, we first calculate the ratio between the left propagating power $P_l$, at the output target waveguide (after the coupler), and the total emission power emitted by the dipole $P_t$; then a mode overlap integration is performed between the destination waveguide mode and the computed propagating electromagnetic fields, whereby the $\beta$ is the product of the overlap integration and the power ratio. Since all of these coupling parameters are not optimized, and given the complexity of single QD coupling, these numbers are already quite impressive. To put these results in context, we have nano-engineered a situation where the probability of emitting one photon with a vacuum wavelength of $\lambda = 1500$ nm, into a standard 200 nm-thick wire waveguide, is more than 60% with a significant Purcell effect.

### 6.2. Single photon source using a PC waveguide, a cavity, and an output coupler

We next propose a device that exploits some of the advantages of integrated cavities, waveguides, and couplers. The proposed single photon source is shown schematically in Fig. 14. The parameters of the two-hole shifted cavity and the waveguide are the same as before. In this case, however, the cavity is now deliberately over-coupled to the PC waveguide; the PC waveguide is then coupled to an output wire waveguide.

Here, the direct 3D FDTD calculation is once more adopted. Although the Purcell factor (see Fig. 15) and the quality factor are significantly reduced in comparison with the unloaded cavity, they are still very large ($F_y \sim 1100$ and $Q \sim 10300$), and are one order of magnitude larger than that of the previous waveguide example; this is because of the increased LDOS at the QD position; although the increased coupling between the cavity and the waveguide leads to the decrease of the Purcell factor compared to the bare cavity, the major advantage is that on-chip emission is possible with the integrated waveguide. The spatial distribution of the field, at the maximum Purcell factor (see Fig. 15a), is shown in Fig. 16, which highlights how the emitted field leaks predominantly into the output waveguide. We also highlight that the peak Purcell factor is certainly larger than that required to enter the strong coupling regime for a typical QD exciton. The analytical forms of the Purcell factors, $\beta$ factor, and an investigation of the strong coupling regime is reported elsewhere [93].

Intuitively, one might imagine that the scattered light from a large-$Q/V_{eff}$ cavity will mainly escape through the vertical decay channel. But here, even with the coupling with the waveguide, the quality factor of the loaded cavity is still high (larger than 10,000); however, surprisingly, the $\beta$ factor is still as high as 50%, which is again without optimization. It can be anticipated that the collection efficiency can be further increased if we increase the coupling between waveguide and cavity, for example, by resizing or reducing the air hole radius at the interface between cavity and waveguide. We also highlight that the frequency of the peak Purcell factor now is $192.7$ THz, which is strategically away from the waveguide band edge. Also, by looking at the band structure, one has a very large bandwidth with which to efficiently couple to the output waveguide propagation mode. For comparison, the no-shift cavity case is also investigated, where the peak Purcell factor is reduced to about 350, which is significantly smaller than that of...
the shifted-hole case; in addition, the factor is only 37%, since the shifted-hole cavity has a much better coupling to the output waveguide mode, and less overlap with radiation modes above the light line; thus a large Purcell factor and a large factor go hand in hand for our designed structure.

7. Input/output relations of a PC waveguide side-coupled to a single-cavity system: single photon switching and electromagnetic induced transparency

Finally, we highlight some unique PC functionalities that can be achieved by side-coupling a cavity to a waveguide. The structure of interest is depicted in Fig. 17. To describe light propagation, we will again use a Green function approach, closely following the works of Cowan and Young [94], and Hughes and Kamada [95]. Although we show an example of just one side coupled cavity, the extension to two coupled cavities is straightforward, and can result in much richer behaviour, e.g., regimes of coupled-cavity-QED [97] and macroscopic QD entanglement [98].

We briefly describe the general formalism, that directly maps on to what quantum optics researchers usually call input/output theory; however, this is what we would perhaps call medium-dependent scattering theory, which has been around far longer than the coined: “input/output theory”. Expanding the transverse Green $G_{\text{w-c}}^T$ of the PC waveguide-cavity system in terms of the cavity and waveguide eigenstates, then $G_{\text{w-c}}^T = \sum_{\alpha,\beta} B_{\alpha\beta} |f_\alpha\rangle \langle f_\beta|$, where $|f_\alpha\rangle$ are the transverse eigenmodes of the separate waveguide and cavity. From the formal definition of $G_{\text{w-c}}^T$, we derive a set of equations in matrix form for the coefficients $B_{\alpha\beta}$. After solving the equation set (by matrix inversion), we obtain the full Green function expression for the waveguide-cavity system:

$$G_{\text{w-c}}^T = G_0^T + \frac{\omega^2 |f_\alpha\rangle \langle f_\alpha|}{\omega^2 - \omega^2 - \omega^2 |V_\alpha G_0^T V_\alpha| |f_\alpha\rangle \langle f_\alpha|} + \frac{\omega^2 |f_\alpha\rangle \langle f_\alpha|}{\omega_0^2 - \omega^2 - \omega^2 |V_\alpha G_0^T V_\alpha| |f_\alpha\rangle \langle f_\alpha|} + \frac{\omega^2 |f_\alpha\rangle \langle f_\alpha|}{\omega_0^2 - \omega^2 - \omega^2 |V_\alpha G_0^T V_\alpha| |f_\alpha\rangle \langle f_\alpha|} + \frac{\omega^2 |f_\alpha\rangle \langle f_\alpha|}{\omega_0^2 - \omega^2 - \omega^2 |V_\alpha G_0^T V_\alpha| |f_\alpha\rangle \langle f_\alpha|}, \quad (24)$$

and since $K = G^T + \sum_\lambda \frac{\sigma_\lambda}{\pi} f_\lambda^*(r) f_\lambda(r')$, we can again set $K_{\text{w-c}} \approx G_{\text{w-c}}^T$ and work exclusively with $G^T$. The shorthand notation of $V_\alpha$, represents the perturbation in the dielectric constant that results from adding in the cavity to the perfect PC lattice [94, 95]. By subsequently exploiting the dipole approximation and the Dyson equation ($G^T = G_0^T + G_0^T V G^T$), we derive the renormalized Green function – including the QD dot response – as

$$\tilde{G}_{\text{w-c}}^T(r, r', \omega) = \frac{G_{\text{w-c}}^T(r, r', \omega)}{1 - n_d \cdot G_{\text{w-c}}^T(r_d, r_d, \omega) \cdot n_d \alpha_0(\omega)}, \quad (25)$$

where $\alpha_0(\omega)$ is again the bare polarizability, as introduced earlier.
Equipped with the Green function of the coupled system, the problem is basically solved: we can immediately derive the input-output relations directly by simple projection. Using the Dyson solution, the total self-consistent electric field operator can be written as, \( \hat{E}(\omega, r) \approx \hat{E}_0(\omega, r) + \int dr' \hat{G}_R^{\text{in}}(r, r', \omega) V_c(r') \hat{E}_0(\omega, r') \), where \( \hat{E}_0(\omega, r) \) is the homogeneous incident-field operator to the waveguide. The field can also be defined directly through the input and output operators: \( \hat{E}(\omega, r) = \sum_{l=L,R} \int d\nu \sqrt{\frac{n_c}{2\pi}} \frac{\hat{a}_{\nu}^{\text{in}}(\omega)}{\nu - \omega} f_l(\nu, r) + \text{H.c.} \), where \( l = L, R \) sums over the left and right output channels along the waveguide. Subsequently, assuming the incident driving field is \( \hat{E}_0(\omega, r) = \int d\nu \sqrt{\frac{n_c}{2\pi}} \frac{\hat{a}_{\nu}^{\text{in}}(\omega)}{\nu - \omega} f_\nu(\nu, r) + \text{H.c.} \), and projecting the Dyson equation onto the eigenmode \( f_R(\nu, r) \) and \( f_L(\nu, r) \), respectively, we obtain the input and output operators:

\[
\begin{align*}
\hat{a}_R^{\text{out}}(\omega) &= \left[ 1 + \frac{i\omega \Gamma_c/[\omega^2 - \omega^2 - i\omega(\Gamma_c + \Gamma_0)]}{1 - n_d \cdot \hat{G}_R^{\text{in}}(\nu, \omega) \cdot n_d \omega_0(\omega)} \right] \hat{a}_{\nu}^{\text{in}}(\omega), \\
\hat{a}_L^{\text{out}}(\omega) &= \frac{i\omega \Gamma_c/[\omega^2 - \omega^2 - i\omega(\Gamma_c + \Gamma_0)]}{1 - n_d \cdot \hat{G}_L^{\text{in}}(\nu, \omega) \cdot n_d \omega_0(\omega)} \hat{a}_{\nu}^{\text{in}}(\omega),
\end{align*}
\]

where \( \Gamma_c = \pi \omega_0^2 |\langle \ell_c | V_c | \ell_f \rangle|^2 \), and \( \Gamma_0 \) is the decay rate of bare cavity. We have assumed the coupling with the left and right waveguide channel is symmetric with respect to the cavity mode. In addition, the emitted field from an excited QD exciton [96] can be obtained by projecting Eq. (9) onto the corresponding output channel, \( f_\nu(\nu, r) \).

In Fig. 18, we show examples of the transmission and reflection, for the systems with and without a coupled QD. The transmission and reflection are defined respectively as:

\[
T = \left| \frac{\hat{a}_R^{\text{out}}(\omega)}{\hat{a}_R^{\text{in}}(\omega)} \right|^2, \quad R = \left| \frac{\hat{a}_L^{\text{out}}(\omega)}{\hat{a}_L^{\text{in}}(\omega)} \right|^2.
\]

The transmission case was first studied by Hughes and Kamada [95], while the transmission case was later studied by Waks et al. [99], but both approaches give essentially identical results. The dipole-induced transparency regime [99, 100] can be clearly seen, which is an effect that originates from the interference between the QD scattered field and the propagating input field. Interestingly, the observed splitting in the regime of dipole-induced transparency, can occur even if we are outside the regime of strong coupling; yet, within the regime of strong coupling, the spectral splitting in transmission or reflection coincides exactly with vacuum Rabi splitting [95]. The theory can also be extended to include other potentially important effects, e.g., the effect of electron-phonon coupling as a function of temperature, which demonstrates the important role of non-Markovian relaxation processes, is reported by Milde et al. [101]. Possible applications of using QD coupled waveguide systems for slow-light buffering have also been reported in [102].

8. Conclusions and Outlook

We have presented a brief review into the growing field of chip-based quantum light sources using planar PCs and single QDs. Since this is a large and expanding research area, we have focused on the basic physics required to design practical single photon sources using such material systems. We believe that physics-design approaches, such as those presented, offer considerably more insight than, for example, “blindly” running large-scale numerical simulations, which is perhaps too-often the current trend these days. Single photon optimization is no longer a matter of positioning a photon emitter near a field antinode, and hoping for the best; rather, key design insights and material functionalities need to be made clear. Systematic and detailed comparisons between experiments and theory are required, with and without coupled QDs. In this regard, we have discussed and applied a unified theoretical framework which can describe, predict, and design a wide variety of practical single photon emitters. A connection to the various experimental and theoretical works appearing in the recent literature has been made. This framework forms the basis for understanding the rich and complex light-matter interactions that can occur in planar PC chips, and can be extended to account for many of the things that we have chosen to leave out, such as coupled QDs, coupled cavities, electron-phonon interactions, input coupling, and entangled photon pair generation [103, 104], to name but a few.

The on-demand single photon source is an extremely important source for quantum information science and technology. As the potential suitable solid-state material of choice, we believe that the prospects for using integrated planar PC chips offer many advantages for controlling and manipulating the light-matter interactions of an embedded QD. In a remarkably small space of time, we have witnessed many impressive experiments, largely performed in an un-deterministic and heroic manner; yet, we have already seen clear evidence that planar PCs do have the ability to mutually control one photon and one electron (or one electron-hole pair) at a time. Aided by improving fabrication techniques, and by new and practical design insights, there are grounds for being optimistic in envisioning what will come next. Furthermore, because on-chip devices
can be encapsulated and reliably produced, the commercialization prospects of single photon sources using semiconductor chips will probably happen within the next five years. Having said that, one must not oversell the dream just yet! Enormous challenges remain, including the understanding and possible control of decoherence processes, the role of fabrication imperfections, as well as the ability to deterministically position single QDs, preferably working with \( \lambda = 1.5 \mu m \), within the chip. Nevertheless, we certainly envision that, in the next few years, the community will realize on-chip single photon light sources using integrated waveguide-cavity systems, such as those presented in this review.

We finish, by congratulating all the experimentalists and theorists working in the field for their continued efforts, and we look forward to see many more exciting developments in the near future.

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References


[93] P. Yao and S. Hughes, Controlled cavity-QED using a planar photonic crystal waveguide-cavity system, submitted; see arXiv:0904.4469v2 [physics.optics].